Pursuing an Idea: Planck's Quantum Theory of Ideal Gas[*]

Massimiliano Badino[†]

1 Prelude: One Hundred Years of 'Quantitude'

Many years later, as he faced the firing squad, Erwin Planck was to remember that distant afternoon when his father took him to discover the quantum. It was indeed in a day at the beginning of the last century, during a walk in the Grunewald Forest, that Max Planck informed his youngest son that his recent discovery was the greatest since the time of Newton. After that day, physics was never the same again. From the mysterious realm of the interaction between matter and radiation, the quantum spread out both in the theory of radiation and in the theory of matter and eventually changed our picture of the physical world. Planck played a foremost role in this revolution till the mid-1920 and especially his work on the application of quantum hypothesis to the ideal gas was instrumental in setting the stage for quantum mechanics.

In this paper I will give a cursory outline of this work and, centering on the argumentative structure, I will argue that Planck's theory of gas was framed according to a theoretical strategy he had already adopted in his previous work on radiation theory. The thrust of this strategy consists in focussing on the general features of the system and leaving aside specific assumptions on the micro-processes. Throughout the years Planck reconfigured and reorganized his arguments and the great flexibility of his theoretical strategy allowed him to maintain a consistent outlook on the problem. The development of Planck's theory of quantum gas can be divided into three phases. Firstly the lecture at the Wolfskehl Conference held in Göttingen in 1913 where Planck presented a sketchy account of the phase equilibrium between gas and condensate. Secondly, after some destructive criticisms against his initial attempt, Planck changed radically the approach and elaborate a more mature theory in 1916 and then in 1921. In the third phase, from 1921 to 1925, Planck was mostly concerned with the defense of his theory. In this paper I will especially focus on a particular problem, namely on the justification of the extensivity term for entropy. This problem will be our key to enter Planck's theory of quantum gas.

2 From the *Wärmestrahlung* to the Wolfskehl Lecture

Thermodynamics and gas theory had been the protagonists of 19th century physics. However, at the turn of the century, many quandaries still remained to solve and new ones sud-

---

denly came out. One of them concerned the calculation of the additive constant of entropy, which in turn was tightly related to the chemical constant.[1] In 1906 Walther Nernst argued that the maximum work that can be obtained from a reaction tends to become equal to the internal energy as the temperature approaches zero (Nernst 1906). In his *Thermodynamik* (Planck 1911), Planck extended this statement — later known as third principle of thermodynamics or Heat Theorem — by claiming that at absolute zero the entropy vanishes.[2]

Unexpectedly, entropy had become an absolute quantity and its additive constant, which 19th century thermodynamics left undetermined, could, in principle be calculated. For usual statistico-mechanical techniques provide for the entropy the following equation (Sommerfeld 1947, 218-219):

$$S = kN\ln(VT^{2/3}) + \frac{2N}{3}\ln\left(\frac{2\pi mek}{G}\right),$$ (1)

where N is the number of molecules, V the volume of the gas, T the temperature, and G the volume of the elementary region of the phase space. Classical statistical mechanics does not say anything about the value of G, but in 1912 Otto Sackur and Hugo Tetrode independently suggested that $G = h^f$ where f is the number of degrees of freedom of the system and h is Planck's constant (Tetrode 1912, Sackur 1911).[3] Promisingly, the resulting chemical constant was found in excellent agreement with empirical data. However, equation (1) does not fulfill the extensivity requirement. In order to have a double entropy in case of doubling the system equation (1) must be replaced by:

$$S = kN\ln(VT^{2/3}) + \frac{2N}{3}\ln\left(\frac{2\pi mek}{NG}\right).$$ (2)

How to justify the modification in the entropy formula was by no means clear. Tetrode suggested a solution that later would become standard: if one subtracts from equation (1) a term $\ln N!$ and assumes Stirling's approximation by virtue of which $N!$ can be replaced by N^N, one obtains exactly equation (2). This would imply to divide probability (or later partition function) by $N!$. The rationale of this division remained however an open question.

Sackur tried to tackle the problem in a different way. He assumed that the elementary region of the phase space depends on the number of molecules. This entailed that the elementary volume G in equation (1) should be replaced by Ng with g new 'elementary volume' independent of N. However Sackur was unable to provide more than an evocative description of the molecular behavior that supposedly lies behind this hypothesis (Sackur 1911, 969):

Die allerdings schwer vorstellbare physikalische Interpretation dieses Ergebnisses würde [...] verlangen, daß sich die Molekeln der Gase nicht im ganzen ihnen zur Verfügung stehenden Raum gleichmäßig verteilen, und nicht alle möglichen Geschwindigkeiten von Null bis unendlich annehmen können, sondern

---

[1]The chemical constant, which is responsible for the chemical equilibrium, is the integration constant of the Gibbs-Helmholtz equation or, in case of phase equilibrium, of the Clausius-Clapeyron equation while the entropy constant is the usual integration constant of the definition of entropy. They are conceptually independent though they differ numerically only by a fixed quantity.

[2]For a more general overview see (Darrigol 1991) and (Desalvo 1992). On the several derivations of the heat theorem see (Kox 2006).

[3]The relation between h and the elementary volume of phase space had been first suggested by Planck in his 1906 *Vorlesungen über die Theorie der Wärmestrahlung* and linked to the concept of equiprobability.

daß sie sich um einzelne Raumpunkte, etwa wie die Schüsse nach einer Scheibe, anhäufen und daß ihre Geschwindigkeitscomponenten sich sprungweise ändern.

Prophetic as they can sound to a modern ear, these words do not amount to a physical argument. A man capable of simultaneously mastering physics and statistics was desperately needed. That man was Planck.

Planck entered the debate on quantum gas theory from the very beginning and his enthusiasm is easy to explain. For in the Sackur-Tetrode's quantum interpretation of the elementary region, Planck immediately envisioned a powerful and far-reaching way of understanding the meaning of the constant h. He was so excited by this train of thought, that he dedicated to the problem of ideal gas a specific chapter in the second edition of his *Vorlesungen über die Theorie der Wärmestrahlung*, written in 1912 but published in 1913. Specifically, Planck stressed that the fixed size of the elementary region is related to the absolute meaning of probability and, in turn, of entropy (Planck 1913, 123):

Daß [eine Elementargebiet von bestimmten Grösse] wirklich existiert, ist der hier entwickelten Theorie im Gegensatz zur Boltzmannschen Theorie eigentümlich und bildet den Inhalt der sogenannten Quantenhypothese. Dieselbe ist, wie man sieht, eine unmittelbare Folgerung aus dem Satze [...], daß der Entropie S ein absoluter Wert zukommt; denn dies bedingt [...] auch einen absoluten Wert für die Größe der thermodynamischen Wahrscheinlichkeit W, welche ihrerseits [...] durch die Anzahl der Komplexionen, also durch die Anzahl und Größe der zugrunde gelegten Elementargebiete bedingt is.

On a more operative front, Planck took up Sackur's suggestion of a dependence of the elementary region on the number of molecules, but he had little to support this endorsement and confined himself to claim this dependence but a vague hint (Planck 1913, 131):

Es versteht sich, daß die Größe g in enger Beziehung stehen muß du dem, einstweilen noch gänzlich unbekannten, Gesetz, nach welchem die Moleküle mikroskopisch aufeinander wirken.

Few mouths after completing the second edition of the *Wärmestrahlung*, Planck had the opportunity of developing more deeply his ideas on the topic. This opportunity was offered by the Wolfskehl Lectures held in Göttingen in April 1913. David Hilbert, eager to put Göttingen on the map of quantum physics, used the famous Wolfskehl fund — initially meant as a prize for proving Fermat's Last Theorem — to organize a grand conference with a battery of the best physicists working in Germany and Holland.[4] In his contribution Planck develops a quantum theory of phase equilibrium between a vapor (supposed an ideal gas) and its condensate (supposed a solid to be treated with Debye's theory) and compares the result with kinetic theory. Most importantly, he fine-tunes a conceptual tool that would be crucial for his investigation on entropy extensivity, namely the dis-analogy between a gas of molecules and a set of oscillators. Famously, in 1900 Planck had obtained the black-body radiation law by combinatorial considerations applied to the entropy of a set of oscillators.

---

[4]The list of participants comprised beside Planck: P. Debye, W. Nernst, M. von Smoluchowski, A. Sommerfeld, H. A. Lorentz, H. Kamerlingh-Onnes, and W. H. Keesom. On the fundamental role played by Hilbert in promoting quantum physics in Göttingen see (Schirrmacher 2003) and (Corry 2004).

It was therefore perfectly natural for him to start from this well-studied case. And the disanalogy is indeed enormously instructive.

Oscillators are bounded to certain points in space, work according to a proper frequency and interact by resonance. This means that, generally speaking, an oscillator can interact with the radiation field, but not with another oscillator of different frequency. By contrast, atoms are free to move and to interact with any other atom by elastic collision or other kind of interaction. This entails that, while an oscillator is represented by a simple elementary region in the phase space defined by its elastic constants, the elementary region of an atom is much more complex and must be defined by taking account of the allocation of the other atoms as well. Hence, Planck claims, an elementary configuration comprises at least two atoms in mutual interaction, so that the elementary region would become a phase space generalization of the usual kinetic notion of 'sphere of action': «Die [Elementargebiet] könnte durch die Eigenschaft charakterisiert sein, daß sie die Grenze angibt zwischen zwei ganz verschiedenen Typen von Bewegungen, die das bewegliche Atom unter der Einwirkung des ruhend gedachten Atoms nach den Gesetzen der Quantendynamik ausführen kann», (Planck 1914, 7-8). Since the motion of the atom is constrained by the spheres of action of the remaining ones and since the motion is confined in the elementary region, it follows that the elementary region G of a gas depends on the number of molecules $G = Ng$ as Sackur had claimed. Planck comments (Planck 1914, 8):

> Hier offenbart sich ein charakteristischer Gegensatz gegen die Verhältnisse bei den Oszillatoren, die in der Theorie der Wärmestrahlung benutzt werden. Diese üben keine direkten Wirkungen aufeinander aus, infolgendessen ist ihr Zustand nicht von ihrer gegenseitigen Lage abhängig, ihre relativen Koordinaten haben keinerlei Einfluß auf die Größe des Elementargebiets, und deshalb ist das Elementargebiet nicht proportional ihrer Anzahl.

Planck's argument is rather obscure. On the one hand the constraint that the remaining atoms exert on the single one seems to have an eminently spatial character, but on the other hand, the elementary region is defined in the phase space, not in the configuration space. This point was perceptively stressed by Hendrik Antoon Lorentz in a review paper entitled "Some remarks on the theory of monoatomic gases". Lorentz argued that if one constructs the elementary region of an atom by joining all elementary regions of the remaining ones, then one ends up with something that is no longer a region in the phase space. For a partition of the phase space must depend on the general features of the system and not on the relative allocation of its elements. Furthermore, Planck's G is not an elementary region of probability either, because the position of the atom in the region will be strictly determined by the others and, Lorentz said: «this can hardly be admitted; whether the N-th molecule will lie near the first or near any other of the molecules that are already present must certainly be considered as something accidentals», (Lorentz 1914, 172).

On the same line was a criticism by Obe Postma, a collaborator of Lorentz's in Leiden. In a paper published in 1915, Postma underlined the statistico-mechanical issues involved in the question and argued that Planck's entropy could not be extensive without making appropriate assumptions. More specifically he claimed that (Postma 1915):

> To make up for this Planck assumes that G is proportional to N, quite arbitrarily in my opinion; the reasoning, namely, by which he tries to make plausible that this "elementary region of probability" would be proportional to N, does

not hold for a perfect gas. It would have been better, it seems to me, to divide the "thermodynamic probability", which does not represent the originally meant probability at all now, also by $N!$, as Tetrode did for the denominator.

Evidently, Planck's argument was not convincing the experts. After these devastating criticisms Planck abandoned the issue of phase equilibrium and returned to the basic elements of his theoretical strategy.

3 Digging into the phase space structure: 1916-1921

Throughout his scientific career Planck followed a general theoretical strategy: to construct theories that were independent of arbitrary assumptions on the microscopic elements. Planck did not like a physics in which the final result hinged heavily on specific hypothesis on the constitution of objects out of the range of our direct experience. He favored much more a physics of general principles in which an all-embracing and flexible conceptual framework leads to empirical results regardless of the constitution of the micro-level.

In his studies on physical chemistry in the 1880s and 1890s, this strategy was substantiated by an application of thermodynamical concepts and tools that hardly involve atoms or molecules. This viewpoint is masterfully exposed in his book on thermodynamics. Similarly, the investigations on heat radiation were shaped according to the same strategy through a careful problem choice. For black-body radiation can be treated as a purely thermodynamical-electromagnetical problem and the final result is independent of the mechanism warranting the equilibrium in the cavity.

Unsurprisingly, this theoretical strategy is at work also in Planck's quantum theory of gas where it is pursued by means of a particular formal procedure that Planck had been developing since 1906 as a generalization of his theory of heat radiation: the analysis of the phase space. Planck's recipe is simple. In his thermodynamics he uses the 'characteristic function:'

$$\Psi = -\frac{F}{T} = k \ln Z, \tag{3}$$

where F is the free energy, k Boltzmann's constant, T the temperature and Z, as usual, the partition function or *Zustandsumme*:

$$Z = \sum e^{-\overline{U}/kT}, \tag{4}$$

where $\overline{U}$ is the average energy of a state and the summation is extended over all accessible states. From the characteristic function other thermodynamic quantities like energy and entropy can be easily attained according to the well-known formulae:

$$U = T^2 \frac{\partial \Psi}{\partial T}, \qquad S = \Psi + \frac{U}{T}. \tag{5}$$

Thus, the thermodynamical description of the system is traced back to counting the accessible states namely to the division of the phase space into elementary regions. This procedure is extremely general and, in fact, it was also applied to atomic physics in the same period of time. Indeed, Planck looked with interest at the development of atomic physics

and especially at Sommerfeld's work, who in those days was finding his way toward a generalization of quantum conditions through a similar path.[5]

The analysis of the phase space as a tool to pursue a general theoretical strategy introduces an important shift in Planck's argumentative line. For the focus moves from the physical objects (atoms understood as mechanical systems) to the ways of representing and formally manipulating the objects themselves. As a consequence, the investigation becomes centered on the properties of our general description of the system more than on the properties of the system. A specific example of this conceptual shift is the way in which Planck handles the problem of entropy extensivity.

After some papers in 1915 and 1916 where he dwelt on the description of the phase space of a mechanical system (Planck 1915a, Planck 1915b, Planck 1916a), Planck faced directly the issue of extensitivity in (Planck 1916b) where he tried to justify the correction term $-N \ln N$ through a division by $N!$ *á la* Tetrode of the partition function. The volume of the n-th elementary region (or quantum state) of a system of N dissimilar atoms is:

$$G_n = (nh)^{3N}. \tag{6}$$

In other words, the elementary region in the phase space of the system is the bare juxtaposition of the elementary regions in the phase spaces of the single atoms. In this case the partition function results immediately from the multiplication of the partition functions of single atoms, but the condition that atoms are dissimilar [ungleichartig] plays a foremost role (Planck 1916b), see (Planck 1958, II, 428):

> Denn nur in diesem Falle entspricht jedem in mikroscopisch genauen Sinne definierten physikalischen Zustand des Körpers ein bestimmter Punkt im Phasenraum. Wenn aber im Körper Gruppen von gleichartigen Atomen enthalten sind, so trifft dies nicht mehr zu, vielmehr ist dann einem bestimmten physikalischen Zustand des Körpers eine mehr oder minder große Anzahl von physikalisch vollkommen gleichbedeutenden Punkten des Phasenraumes zugeordnet, da ein bestimmter Punkt des Phasenraumes für jedes einzelne Atomindividuum bestimmte Koordinate und Geschwindigkeiten bedingt. Soviel Permutationen also die gleichartigen Atome zulassen, ebensoviel Phasenpunkte entsprechen einem bestimmten physikalischen Zustand. Zur bequemeren Ausdruckweise will ich daher zwischen "Phasenpunkt" und "Zustandpunkt" unterscheiden.

All permutations of points in the *Phasenraum* give the same point in the *Zustandraum* therefore the region calculated by simple multiplication is $N!$ times larger than the physically meaningful region:

$$G_n = N!(nh)^{3N}. \tag{7}$$

This new partition of the phase space provides the correction term for extensivity. Planck's argument could resemble the classical combinatorial procedure in which the physical distribution is obtained by permuting complexions. But the state of affairs, apart from the superficial resemblance, is much more complex. Planck is not claiming that the division is necessary because we change the description of the system and move from its detailed microstate to its measurable corse-grain state. On the contrary both spaces are descriptions

---

[5]On this issue see Michael Eckert's contribution to this volume.

of the microstate of the system as it is evidenced by the quotation above. The difference, hence, is more subtle and tricky.

Planck understands the *Phasenraum* as a mental space in which one can perform purely formal operations of combination and permutation of objects. These operations are completely independent of the nature of the objects. However, in the physical microstate what matters is the "localization of momenta", that is the coupling between one coordinate and its conjugate momentum, whereas the individuality of atoms is completely immaterial. Thus, while in the *Phasenraum* we can mentally manipulate individual atoms, a point in the *Zustandraum* is given by a couple coordinate-momentum regardless of *which atom* is occupying this or that position with this or that momentum. Planck clarified this fundamental point much later in 1925 (Planck 1925), (Planck 1958, 600):

> Nun ist ein Zustand eines aus gleichartigen Molekülen bestehenden Gases vollständig bestimmt, wenn man erstens alle Raumpunkte kennt, in denen sich ein Molekül befindet, und zweitens die Geschwindigkeit des Moleküls. Man braucht aber nicht zu wissen, "welches" Molekül ist. Falls man also zur Berechnung der Gesamtzahl der in dem Phasengebiet enthaltenen Zustände für jedes Molekül über alle in Betracht kommenden Raumpunkte summiert, so erhält man jeden Zustand des Gases nicht einmal, sondern $N!$ mal.

For various reasons, not in the least the war, Planck did not publish anything on quantum theory of gas in the next five years. In 1921 he returned to the topic on occasion of the fourth edition of the *Wärmestrahlung*. Again, and even more strongly, the starting point is the dis-analogy with oscillators. In particular, Planck claims that mutual interactions between atoms force us to forgo a sharp definition of microstate and to resort to the description of the system *as a whole* by means of Gibbs' statistical mechanics.[6] In the fifth section of the 1921 *Wärmestrahlung*, Planck states that the thermodynamic equations derived from the usual combinatorial techniques, «gelten [...] für N Atome in einem gemeinsamen Volumen V, vorausgesetzt, daß diese Atome, trotz ihrer gleichen Masse, alle verschiedenartig sind», and immediately after he claims that the misunderstanding between statistical description for similar and dissimilar atoms hinges on the fact that «die N Atome unabhängiger voneinander angenommen wurden», (Planck 1921b, 205). A description of the system in terms of simple juxtaposition of its elements is impossible and must be replaced by the brute given of a complex system of $3N$ degrees of freedom.

Thus, in 1921 Planck leaves temporarily aside the thorny difference between *Phasenraum* and *Zustandraum*, but not the supporting idea: the dissimilarities between atoms and oscillators. Oscillators are bounded to some positions in space so they can be easily localized, although they are physically similar. In a system of oscillators, the energy can be distributed over space regions that are mutually separate and independent: «die Oszillatoren bzw. Rotatoren sind dauernd getrennt und daher individuell jederzeit unterscheidbar», (Planck 1921a, 370). On the contrary, atoms fly here and there colliding with one another and this originates the *Austauschmöglichkeit*, the possibility of exchanging atomic state without changing the (micro)description of the system as a whole. Therefore, Planck claims, one has to abandon a detailed description of the individual configuration, and to deal with the system as a whole by virtue of its internal symmetries (Planck 1921b, 209):

---

[6] On this point see especially (Darrigol 1991).

Denn da die Atome alle gleichartig sind, so sind zwei Zustände des Gases, welche sich nur durch Orts- und Geschwindigkeitswechsel zweier oder meherer Atome unterscheiden, in physikalischer Hinsicht vollkommen identisch. Man beachte den grundsätzlichen Unterschied gegen den früher behandelten Fall von Oszillatoren oder Rotatoren. Dort waren die Moleküle vollständig voneinander getrennt, jedes hatte seinen bestimmten dauernden Platz, und wenn zwei Moleküle ihre Energien austauschen, so ergab dies eine neue Komplexion. Hier dagegen haben wir kein System voneinander getrennter Moleküle, sondern ein einziges mit Symmetrien ausgestattetes Gebilde, und diese Symmetrien bestehen darin, daß es kein physikalisches Kennzeichen gibt, welches ein bestimmtes Atom wiederzufinden gestattet, wenn man das Gas erst in einem, dann in einem anderen Zustand betrachten. Daher bedingen zwei nur durch Umstellung zweier Atome unterschiedene Zustände des Gases keine neue Komplexion.

All this calls for a modification of the combinatorial description used in classical statistical mechanics. The partition function Z_G for the gas as a whole must be calculated from the partition function of a set of independent objects *via N*! division:

$$Z_G = \frac{Z^N}{N!},$$ (8)

where Z is the partition function for a single atom. Thus the correction term for extensivity is eventually justified by combination of very general arguments concerning the modes of description of system and some physical constraints on this description. No room is left in Planck's picture for specific hypothesis on the nature of micro-objects.

4   Defending the theory (1921-1925)

Planck's defense of his theory of quantum gas against criticisms and objections was strenuous. As early as 1921, Paul Ehrenfest and Viktor Trkal argued that Planck's application of the formalism of statistical mechanics, and in particular of Boltzmann's definition of entropy, to individual states was illegitimate. Moreover, they insisted that extensivity concerns the difference of entropies between states connected by reversible transformations rather than individual state (Ehrenfest and Trkal 1921). Planck replied almost immediately stating that his application of Boltzmann's entropy was of great practical usefulness and adding that «braucht man nach meiner Mainung sich nicht den Kopf zu zerbrechen über die Bedeutung einer Größe für einen Prozeß, der in der Natur gar nicht vorkommt, sondern man kann sich damit begnügen, die Entscheidung über die Brauchbarkeit dieser Größe davon abhängig zu machen, ob ihr theoretisch berechneter Wert für alle diejenigen Prozesse, welche wirklich beobachtet werden, mit dem gemessenen Wert übereinstimmt oder nicht», (Planck 1921a), (Planck 1958, II, 529). This pragmatic attitude is not surprising: as said above, Planck's theory concerns modes of description of the physical objects more than the objects *per se*, therefore pragmatism seems here much more appropriate than realism.

To most commentators Planck's theory was a seemingly natural modification of the classical way of counting states, but, on the other hand it was difficult to understand Planck's rationale for such a modification. This was due to the peculiarity of Planck's argumentative line. At times Planck presents the *N*! division as a purely formal consequence of a new combinatorial procedure: «die Division mit *N*! [...] erweist sich dann und nur dann als

notwendig, wenn man, statt, wie vorgeschrieben ist, über alle versciedenen Elementargebiete des Gebildes, nämlich des Gasmodells, zu summieren, über alle Elementargebiete jedes einzelnen Atoms summiert», (Planck 1921a), (Planck 1958, II, 533). However, it would be wrong to regard the *N*! division as a pure statistical *Ansatz*. In his excellent analysis of the problem, Erwin Schrödinger suggested exactly this view and claimed that Planck's *N*! could be interpreted as the internal symmetry number of a complex structure (Schrödinger 1925, 438-439). Planck protested against this interpretation and stressed that the modification in the combinatorial description stemmed from the fact that «die Moleküle in Lauf ihrer Bewegungen ihre Plätze tauschen können und daher in zwei verschiedenen Zustände des Gases nicht paarweise zu identifizieren sind», (Planck 1925), (Planck 1958, II, 600-601).[7]

The solution of the dilemma lies in the complicate relation between the combinatorial nature of the *N*! division and its physical grounds. Planck uses very general physical features of the system, like the dis-analogy between atoms and oscillators, to impose some constraints on the formal combinatorial scheme but he eschews detailed assumptions on the mechanism of interaction. Consequently, Planck's justification of the extensivity term hinges on some global property of the system under investigation, but does not imply any conclusion on the behavior of the particles or, more generally, on the constitution of the microlevel. This very high-level kind of justification of a combinatorial operation regarding physical system is what Planck strove for from 1913.

This is one of the reasons why Planck did not arrive at Bose-Einstein statistics and did not develop a concept of indistinguishable particles in modern sense. He indeed discusses Gibbs' paradox and the conditions of distinguishability or indistinguishability (Planck 1922), but his answer is limited to energetic states. As in his 1916 paper, distinguishability or indistinguishability are not features of the objects, but of our modes of description of them: what matters for such description is the localization or the allocation of energy, therefore particles with different (internal or kinetic) energy are distinguishable. The issue of mutual statistical dependence between atoms never arose and only the permutability on the phase space was considered. In completely accordance with his theoretical strategy, any question about supposed new features of the microlevel remained out of Planck's perspective.

5   Conclusions

It would be easy, in hindsight, to dismiss Planck's solution to the problem of extensivity as a bunch of missed opportunities. However, if not from a scientific point of view, the conceptual trajectory of his investigation on quantum gas has much to offer to the historian of science. In particular, it shows us how a clear theoretical strategy, not necessarily epistemologically committed, can be continually reconfigured by means of different approaches, problem choices, and formal procedures. Moreover, it shows us how deep was the entanglement between the ways of counting objects (statistics) and the objects counted (physics) in quantum theory of gas. Admittedly, Planck's account experienced a constant tension between a general outlook and the demand for physical content. For physical features of the system were preferably understood as constraints on the combinatorics of the phase space, rather than commitments on the behavior of micro-objects. Ultimately, Planck was not able to untying the conceptual knot concerning *N*!: a purely combinatorial requirement or the

---

[7]On Schrödinger's analysis of Planck's theory see also (Hanle 1977) and (Darrigol 1992).

mark of a new microreality. After all it was more consistent with his scientific history to evade the question.

References

Corry, L.: 2004, *David Hilbert and the Axiomatization of Physics (1898-1918)*, Archimedes, Kluwer Academic Publisher, Dordrecht.

Darrigol, O.: 1991, Statistics and combinatorics in early quantum theory, II: Early symptoma of indistinguishability and holism, *Historical Studies in the Physical Sciences* **21**, 237–298.

Darrigol, O.: 1992, Schrödinger's statistical physics and some related themes, *in* M. Bitbol and O. Darrigol (eds), *Erwin Schrödinger. Philosophy and the Birth of Quantum Mechanics*, Editions Frontieres, Singapore, pp. 237–276.

Desalvo, A.: 1992, From the chemical constant to quantum statistics: a thermodynamic route to quantum mechanics, *Physis* **29**, 465–537.

Ehrenfest, P. and Trkal, V.: 1921, Ableitung des dissociationgleichgewichts aus der quantentheorie und darauf beruhende berechnung chemischer konstanten, *Annalen der Physik* **65**, 609–628.

Hanle, P.: 1977, The coming of age of Erwin Schrödinger: His quantum statistics of ideal gas, *Archive for the History of Exact Sciences* **17**(2), 165–192.

Kox, A. J.: 2006, Confusion and clarification: Albert Einstein and Walther Nernst's Heat Theorem, 1911–1916, *Studies in History and Philosophy of Modern Physics* **37**, 101–114.

Lorentz, H. A.: 1914, Some remarks on the theory of monoatomic gases, *Proceedings Koninklijke Akademie van Wetenschappen te Amsterdam* **19**, 737–751.

Nernst, W.: 1906, Über die Berechnung chemischer Gleichgewichte aus thermischen Messungen, *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse* **1**, 1–39.

Planck, M.: 1911, *Vorlesungen über Thermodynamik*, 3 edn, Barth, Leipzig.

Planck, M.: 1913, *Vorlesungen über die Theorie der Wärmestrahlung*, 2 edn, Barth, Leipzig.

Planck, M.: 1914, Die gegenwärtige Bedeutung der Quantenhypothese für die kinetische Gastheorie, *in* D. Hilbert (ed.), *Vorträge über die kinetische Theorie der Materie und der Elektrizität*, Teubner, Göttingen.

Planck, M.: 1915a, Die Quantenhypothese für Molekeln mit mehreren Freiheitsgraden. I Mitteilung, *Verhandlungen der Deutschen Physikalischen Gesellschaft* **17**, 407–418.

Planck, M.: 1915b, Die Quantenhypothese für Molekeln mit mehreren Freiheitsgraden. II Mitteilung, *Verhandlungen der Deutschen Physikalischen Gesellschaft* **17**, 438–451.

Planck, M.: 1916a, Die physikalische Struktur des Phasenraumes, *Annalen der Physik* **50**, 385–418.

Planck, M.: 1916b, Über die absolute Entropie einatomiger Körper, *Sitzungsberichte der preussischen Akademie der Wissenschaft* pp. 653–667.

Planck, M.: 1921a, Absolute Entropie und chemische Konstante, *Annalen der Physik* **66**, 365–372.

Planck, M.: 1921b, *Vorlesungen über die Theorie der Wärmestrahlung*, 4 edn, Barth, Leipzig.

Planck, M.: 1922, Über die freie Energie von Gasmolekülen mit beliebiger Geschwindigkeitsverteilung, *Sitzungsberichte der preussischen Akademie der Wissenschaft* pp. 63–70.

Planck, M.: 1925, Über die statistische Entropiedefinition, *Sitzungsberichte der preussischen Akademie der Wissenschaft* pp. 442–451.

Planck, M.: 1958, *Physikalische Abhandlungen und Vorträge*, Vieweg und Sohn, Braunschweig.

Postma, O.: 1915, Entropy and probability, *Proceedings Koninklijke Akademie van Wetenschappen te Amsterdam* **18**, 1048–1056.

Sackur, O.: 1911, Die Anwendung der kinetischen Theorie der Gase auf chemische Probleme, *Annalen der Physik* **36**, 958–980.

Schirrmacher, A.: 2003, Planting in his neighbor's garden: David Hilbert and early Göttingen quantum physics, *Physics in Perspective* **5**(1), 4–20.

Schrödinger, E.: 1925, Bemerkungen über die statistische Entropiedefinition beim idealen Gas, *Sitzungsberichte der preussischen Akademie der Wissenschaft* pp. 434–441.

Sommerfeld, A.: 1947, *Vorlesungen über theoretische Physik*, Vol. II, 2 edn, D.V.B., Wiesbaden.

Tetrode, H.: 1912, Die chemische Konstante der Gase und das elementare Wirkungsquantum, *Annalen der Physik* **38**, 434–442.