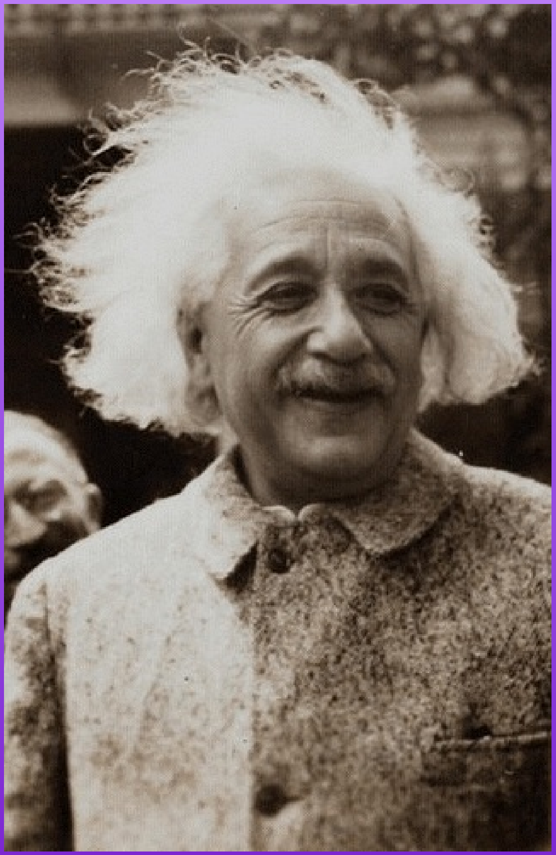
EPR and the Hole Argument:

In What Sense Did Einstein Think Quantum Mechanics Incomplete?



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Connections between Relativity and Quantum Theory in Einstein's Work

Two connections I've explored before:

1. Separability, Locality, Entanglement, and Quantum Theory

2. General Covariance and Bose-Einstein Statistics

Another connection to be discussed today:

3. Conceptions of Completeness in Physical Theory

The Original 1935 EPR Argument

Einstein, Podolsky, and Rosen. “Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?” *Physical Review* 47 (1935), 777-80.

Completeness Condition:

Every element of the physical reality must have a counterpart in the physical theory.

Criterion of Physical Reality:

If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

of lanthanum is 7/2, hence the nuclear magnetic moment as determined by this analysis is 2.5 nuclear magnetons. This is in fair agreement with the value 2.8 nuclear magnetons determined from La III hyperfine structures by the writer and N. S. Grace.[9]

[9] M. F. Crawford and N. S. Grace, Phys. Rev. **47**, 536 (1935).

This investigation was carried out under the supervision of Professor G. Breit, and I wish to thank him for the invaluable advice and assistance so freely given. I also take this opportunity to acknowledge the award of a Fellowship by the Royal Society of Canada, and to thank the University of Wisconsin and the Department of Physics for the privilege of working here.

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Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

1.

ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one

What Concept of Completeness is Intended by EPR?

Syntactic Completeness:

Every wff (well-formed-formula) or its negation
is a theorem.

Semantic Completeness:

Every truth under the intended interpretation
is a theorem.

(All of this is commonplace by 1935.)

The EPR notion of completeness looks somewhat
like the notion of semantic completeness.

Einstein and Gödel at the IAS

But, as we all know, Einstein did not write the EPR paper and did not like the way it turned out.

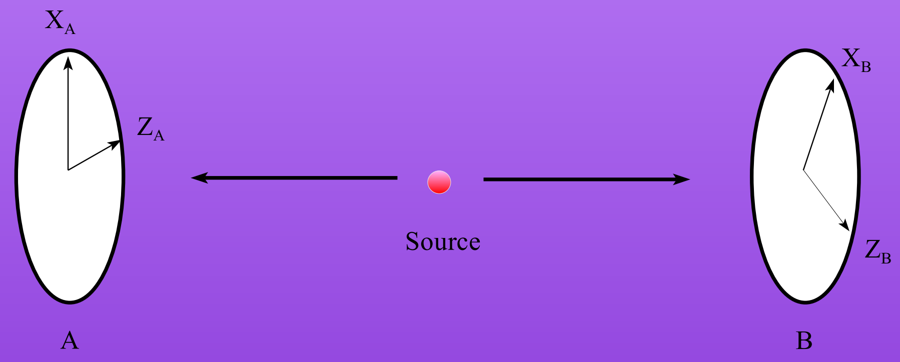
Einstein to Erwin Schrödinger, 19 June 1935

I was very pleased with your detailed letter, which speaks about the little essay. For reasons of language, this was written by Podolsky after many discussions. But still it has not come out as well as I really wanted; on the contrary, the main point was, so to speak, buried by erudition [die Hauptsache ist sozusagen durch Gelehrsamkeit verschüttet].

See: Don Howard. "Einstein on Locality and Separability." *Studies in History and Philosophy of Science* 16 (1985), 171-201.

Einstein's Own Argument for the Incompleteness of Quantum Mechanics

Separability + Locality ⇒ Incompleteness



1. *Separability Principle:* Spatiotemporally separated systems possess their own separate, individual real physical states, of such a kind that the composite state of a joint system is wholly determined by the separate states of the component systems.

2. *Locality Principle:* The real physical state of a system in one region of spacetime cannot be influenced by events in a region of spacetime separated from the first by a spacelike interval. (No action at a distance.)

   Thus, there exists a single real state of affairs regarding the system in the B wing, regardless of what observable one chooses to measure in the A wing.

   But quantum mechanics assigns different theoretical states (different ψ-functions) to system B depending upon the choice of an observable to measure in the A-wing.

3. Hence – Quantum mechanics is incomplete.

Einstein’s Own Conception of Incompleteness

Einstein to Schrödinger, 19 June 1935.

In the quantum theory, one describes a real state of a system through a normalized function, Ψ, of the coordinates (of the configuration-space). . . . Now one would like to say the following: Ψ is correlated one-to-one with the real state of the real system. . . . If this works, then I speak of a complete description of reality by the theory. But if such an interpretation is not feasible, I call the theoretical description “incomplete.”

Einstein’s Own Conception of Incompleteness

Albert Einstein. “Physics and Reality.” *Journal of the Franklin Institute* 221 (1936), 349-82.

Consider a mechanical system consisting of two partial systems A and B which interact with each other only during a limited time. Let the Ψ function before their interaction be given. Then the Schrödinger equation will furnish the Ψ function after the interaction has taken place. Let us now determine the physical state of the partial system A through a measurement which is as complete as possible. Then quantum mechanics allows us to determine the Ψ function of the partial system B from the measurements made, and from the Ψ function of the total system. This determination, however, gives a result which depends upon *which* of the state variables of A have been measured (for instance, coordinates or momenta). Since there can be only *one* physical state of B after the interaction, which state cannot reasonably be considered to depend upon the kinds of measurements I carry out on the system A separated from B, it is thus shown that the Ψ function is *not* unambiguously correlated with the physical state. This correlation of several Ψ functions to the same physical state of system B shows again that the Ψ function cannot be interpreted as a (complete) description of a physical state (of an individual system).

Einstein’s Own Conception of Incompleteness

Albert Einstein. “Autobiographical Notes.” In *Albert Einstein: Philosopher-Scientist*. Paul Arthur Schilpp, ed. The Library of Living Philosophers, vol. 7. Evanston: The Library of Living Philosophers, 1949.

Now it appears to me that one may speak of the real state of the partial system S_2. To begin with, before performing the measurement on S_1, we know even less of this real state than we know of a system described by the Ψ-function. But on *one* assumption we should, in my opinion, unconditionally hold fast: The real situation (state) of system S_2 must be independent of what is done with system S_1, which is spatially separated from the former. According to the type of measurement I perform on S_1, I get, however, a very different Ψ_2 for the second partial system. (Ψ_2, Ψ'_2, . . .) But now the real state of S_2 must be independent of what happens to S_1. Thus, different Ψ-functions can be found (depending on the choice of the measurement on S_1) for the same real state of S_2. (One can only avoid this conclusion either by assuming that the measurement on S_1 changes (telepathically) the real state of S_2, or by generally denying independent real states to things which are spatially separated from one another. Both alternatives appear to me entirely unacceptable.)

If now the physicists *A* and *B* accept this reasoning as sound, then *B* will have to give up his position that the Ψ-function is a complete description of a real situation. For in this case it would be impossible that two different types of Ψ-functions could be correlated with the same situation (of S_2).

Einstein’s Own Conception of Incompleteness

The general principle seems to be:

A unique reality must be given a univocal or unequivocal theoretical description.

What Concept of Completeness Did Einstein Have in Mind?

Eindeutigkeit, Monomorphism, Categoricity

Eindeutigkeit in Physics and the Philosophy of Science

- Joseph Petzoldt. "Das Gesetz der Eindeutigkeit" *Vierteljahrsschrift für wissenschaftliche Philosophie und Soziologie* 19 (1895), 146-203.

- Ernst Mach, Paul Volkmann, Paul Natorp, Ernst Cassirer, and Moritz Schlick on "*Eindeutigkeit*."

- David Hilbert, Causality, and the Cauchy Problem.

See: Don Howard. "Einstein and *Eindeutigkeit*: A Neglected Theme in the Philosophical Background to General Relativity." In *Historical Studies in General Relativity*. Jean Eisenstaedt and A. J. Kox, eds. Boston: Birkhäuser, 1991, 154-243.

Don Howard. "Relativity, *Eindeutigkeit,* and Monomorphism: Rudolf Carnap and the Development of the Categoricity Concept in Formal Semantics." In *Origins of Logical Empiricism*. Ronald N. Giere and Alan Richardson, eds. Minneapolis and London: University of Minnesota Press, 1996, 115-64.

Eindeutigkeit, Monomorphism, Categoricity

Eindeutigkeit in Physics and the Philosophy of Science

Moritz Schlick. “Das Wesen der Wahrheit nach der modernen Logik.” *Vierteljahrsschrift für wissenschaftliche Philosophie und Soziologie* 34 (1910), 386-477.

Truth as univocal coordination between proposition and fact or theory and world.

Eindeutigkeit, Monomorphism, Categoricity

Eindeutigkeit in Physics and the Philosophy of Science

Ernst Cassirer. *Substanzbegriff und Funktionsbegriff. Untersuchungen über die Grundfragen der Erkenntniskritik.* Berlin: Bruno Cassirer, 1910.

Revise Kant by eliminating reliance on intuition, restoring epistemic contact with the world (as presented) in its particularity by accumulating sufficient conceptual determinations (theoretical principles) so as to constrain the object of cognition up to the point of uniqueness or, at least, isomorphism.

Eindeutigkeit, Monomorphism, Categoricity

Categoricity in Mathematics

- David Hilbert, Oswald Veblen, (John Dewey), Adolf Fraenkel, et al. on categoricity:

  A formal theory is categorical if its axioms constrain the class of possible models up to isomorphism.

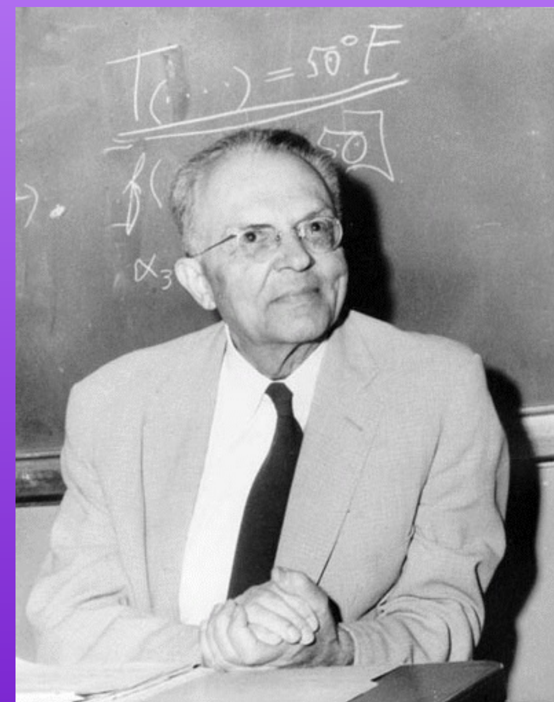
- Noncategoricity is a corollary of Gödel incompleteness for first-order theories.

Eindeutigkeit, Monomorphism, Categoricity

Physics and Mathematics Converge

- Rudolf Carnap, “Eigentliche und Uneigentliche Begriffe.” *Symposion. Philosophische Zeitschrift für Forschung und Ausprache* 1 (1927), 355-374.

- Hermann Weyl. *Philosophie der Mathematik und Naturwissenschaft*. Munich and Berlin: R. Oldenbourg, 1927.



Eindeutigkeit, Monomorphism, Categoricity

Hermann Weyl. *Philosophie der Mathematik und Naturwissenschaft*. Munich and Berlin: R. Oldenbourg, 1927.

An axiom system is complete, or *categorical*, if any two concrete interpretations of it are necessaryily isomorphic. . . . A science can only determine its domain of investigation up to an isomorphic mapping. In particular it remains quite indifferent as to the "essence" of its objects. . . . The idea of isomorphism demarcates the self-evident insurmountable boundary of cognition.

See: Iulian Toader. "Hermann Weyl's Symbolic Constructivism." Ph.D. Dissertation. University of Notre Dame (in progess).

What Does This Have to Do with General Relativity?

The Eindeutigkeit Principle in General Relativity

Eindeutigkeit in the *Lochbetrachtung* or Hole Argument

Albert Einstein and Marcel Grossmann. *Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation. I. Physikalischer Teil von Albert Einstein. II. Mathematischer Teil von Marcel Grossmann.* Leipzig and Berlin: B.G. Teubner, 1913. Reprinted with added "Bemerkungen," *Zeitschrift für Mathematik und Physik* 62 (1914), 225-261.

The Condition on Unique Solutions:

In an acceptable general theory of relativity, the field equations plus boundary conditions must determine a unique solution inside the hole, where $T_{\mu\nu} = 0$.

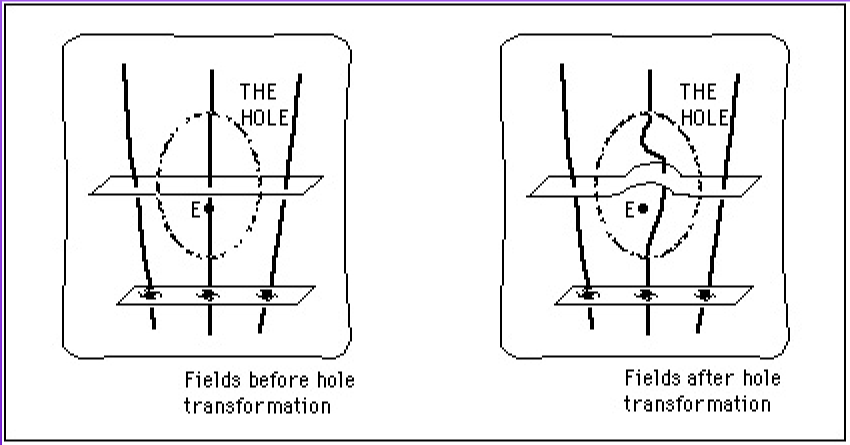
The Eindeutigkeit Principle in General Relativity

Eindeutigkeit in the *Lochbetrachtung* or Hole Argument

Albert Einstein. “Die formale Grundlage der allgemeinen Relativitätstheorie.” *Königlich Preussische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1914), 1030-1085.

Processes [das Geschehen] *in the gravitational field cannot be determined uniquely* [eindeutig festgelegt] *by means of generally-covariant differential equations for the gravitational field*.

If we demand, therefore, that the course of events [der Ablauf des Geschehens] in the gravitational field be completely determined [vollständig bestimmt] by means of the laws that are to be established, then we are obliged to restrict the choice of the coordinate system.



From: John Norton, “The Hole Argument.” Stanford Encyclopedia of Philosophy

The Eindeutigkeit Principle in General Relativity

Eindeutigkeit in the *Lochbetrachtung* or Hole Argument

Einstein to Paul Ehrenfest, probably late fall of 1913.

The questions regarding the theory of gravitation that were still undecided in the summer have clarified themselves in the meantime. A unique determination [eindeutige Bestimmung] of the $g_{\mu\nu}$ out of the $T_{\mu\nu}$ is possible only with the choice of special coordinate systems (rigorously provable).

The Eindeutigkeit Principle in General Relativity

Eindeutigkeit in the *Lochbetrachtung* or Hole Argument

Einstein to Ludwig Hopf, 2 November 1913.

I am now quite satisfied with the theory of gravitation. The fact that the equations of gravitation are not generally covariant, which troubled me inordinately a little while ago, has turned out to be unavoidable; it can easily be proven that a theory with generally covariant equations cannot exist in case it is demanded that the field be mathematically completely determined [vollständig bestimmt] by the matter.

The Eindeutigkeit Principle in General Relativity

Eindeutigkeit in the Point-Coincidence Argument

Einstein to Paul Ehrenfest, 26 December 1915.

From the fact that the two systems $G(x)$ and $G'(x)$, referred to the same reference system [the same x], satisfy the conditions of the grav. field, no contradiction follows with the uniqueness of processes [Eindeutigkeit des Geschehens]. That which was apparently compelling in these reflections founders immediately, if one considers that

1) the reference system signifies nothing real
2) that the (simultaneous) realization of two different g-systems (or better, two different grav. fields) in the same region of the continuum is impossible according to the nature of the theory.

. . . . The physically real in the universe [Weltgeschehen] (in contrast to that which is dependent upon the choice of a reference system) consists *in spatiotemporal coincidences*.* Real are, e.g., the intersections of two different world lines, or the statement that they *do not* intersect. Those statements that refer to the physically real therefore do not founder on any univocal coordinate transformation. If two systems of the $g_{\mu\nu}$ (or in general the variables employed in the description of the world) are so created that one can obtain the second from the first through mere spacetime transformation, then they are completely equivalent. For they have all spatiotemporal point coincidences in common, i.e., everything that is observable.

These reflections show at the same time how natural the demand for general covariance is.

*) and in nothing else!

The Eindeutigkeit Principle in General Relativity

Eindeutigkeit in the Point-Coincidence Argument

Joseph Petzoldt. “Die Relativitätstheorie im erkenntnistheoretischer Zusammenhange des relativistischen Positivismus.” *Deutsche Physikalische Gesellschaft. Verhandlungen* 14 (1912), 1055-1064.

The task of physics becomes, thereby, the univocal [eindeutige] general representation of events from different standpoints moving relative to one another with constant velocities, and the univocal setting-into-relationship of these representations. Every such representation of whatever totality of events must be univocally mappable onto every other one of these representations of the same [1)] events. The theory of relativity is one such mapping theory. What is essential is that univocal connection. Physical concepts must be bent to fit for its sake. We have theoretical and technical command only of that which is represented univocally by means of concepts.

[1)] Better: representations of events in arbitrarily many of those systems of reference that are univocally mappable onto one another are representations of “the same” event. Identity must be defined, since it is not given from the outset.

Other Places Where Worries about Categoricity and Eindeutigkeit Are Relevant to Quantum Mechanics

Stone-von Neumann Theorem

Unitary equivalence of the representations of algebraic quantum mechanics.

Marshall Harvey Stone. "Linear Transformations in Hilbert space, III: Operational Methods and Group Theory." *Proceedings of the National Academy of Sciences* 16 (1930), 172-175.

John von Neumann. "Die Eindeutigkeit der Schrödingerschen Operatoren." *Mathematische Annalen* 104 (1931), 570-578.

Still Other Conceptions of Completeness

Closed Theories and Vollkommenheit:

Alisa Bokulich. “Open or Closed? Dirac, Heisenberg, and the Relation between Classical and Quantum Mechanics.” *Studies in History and Philosophy of Modern Physics* 35 (2004), 377- 96.

“Brandon Fogel. “Epistemology of a Theory of Everything: Weyl, Einstein, and the Unification of Physics.” Ph.D. Dissertation. University of Notre Dame, 2007.