Schrödinger’s Way to Wave Mechanics

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Project

Tip from an ex-historian

Pour étudier l'histoire d'une science particulière, [...] il faut en effet connaître à fond l'état présent de cette science et toutes les péripéties qui l'ont peu a peu amenée à être ce qu'elle est aujourd'hui [...]  

Louis de Broglie

Joint effort to understand the early history of wave mechanics.

- Tracing back the central ideas of wave mechanics.
- Understanding the contemporary debates and questions.
- Analysis of Schrödinger’s publications and notebooks.
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Disclaimer: This is not a finished product!

This presentation:

- Brief overview of the intellectual context.
- Questions guiding our work.
- Samples of Schrödinger materials.
Historical Overview

Multiple roots of wave mechanics

- light quantum hypothesis
- gas statistics
- atomic theory
- optical-mechanical analogy
- matrix mechanics?

Not a root: Direct experimental hints to wave properties to matter.

Rather, paradoxically, experimental hints for light quanta.

No linear story here!

If one dimension is too small, let’s try two dimensions:
## Diagrammatic History

<table>
<thead>
<tr>
<th>Formalism</th>
<th>Light Quantum Statistics</th>
<th>Gas Statistics</th>
<th>Particle Mechanics</th>
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</table>
| Quantum conditions
  Hamiltonian formalism | Planck law
  Light quantum hypothesis | Nernst law
  Planck's quantum statistics | Bohr model |
| 1924 de Broglie mechanical-optical analogy | 1923 de Broglie light quanta-light waves duality | 1923/24 de Broglie matter waves quantum condition |
| 1924 Bose light quantum statistics | | 1924/25 Einstein gas statistics |
| 1925 Landé light quanta superposition | 1924/25 Schrödinger gas statistics | | 1925/26 Schrödinger Wave mechanics Field equation |
| 1925/26 Schrödinger “The old Hamiltonian analogy” | | | |
Diagrammatic History

Formalism
Quantum conditions
Hamiltonian formalism

Light Quantum Statistics
Planck law
Light quantum hypothesis

Gas Statistics
Nernst law
Planck's quantum statistics

Particle Mechanics
Bohr model

1924 de Broglie
mechanical-optical analogy

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light quanta-light waves duality

1924 Bose
light quantum statistics

1924/25 Einstein
gas statistics

1924/25 Schrödinger
gas statistics

1925 Landé
light quanta superposition

1924/25 Schrödinger
Wave mechanics
Field equation

1925/26 Schrödinger
“The old Hamiltonian analogy”
Most of the roots of wave mechanics lie in speculations about the nature of light quanta, in the context of widely diverging programs.

In the early twenties, there was a transfer of “wave” and “particle” pictures between domains (light quantum statistics, gas statistics, atomic theory).

There is a central role of statistical mechanics in the transfer of these pictures.
Schrödinger’s “Zur Einsteinschen Gastheorie” is really Schrödinger’s first paper on wave mechanics.
Schrödinger: Derivation of Einstein’s energy spectrum for the ideal gas from the assumption that gas is a collection of harmonic oscillators. No single molecule states: instead of $n$ molecules having energy state $s$, the oscillator $s$ is in the $n$-th excitation state.

Schrödinger’s “Umdeutung” paper!
<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
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<tbody>
<tr>
<td>1910, 10/12</td>
<td>Peter Debye, „Der Wahrscheinlichkeitsbegriff in der Theorie der Strahlung.“ Derives Planck law from quantization of field oscillations. (Model for Schrödinger's gas theory.)</td>
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<tr>
<td>1918-22</td>
<td>Schrödinger notebooks on “Tensoranalytische Mechanik” dealing with Hamilton-Jacobi formalism. Third notebook: “Analogien zur Optik” (optical-mechanical analogy).</td>
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<tr>
<td>1921/22</td>
<td>Einstein thinks about a dual (wave + particle) theory of light and proposes experiments to test the particle or wave nature of light.</td>
</tr>
<tr>
<td>1921, 5/2</td>
<td>Mieczyslaw Wolfke „Einsteinsche Lichtquanten und räumliche Struktur der Strahlung.“ Derives Planck law from statistics of independent light quanta, using statistically independent n-tuples.</td>
</tr>
<tr>
<td>1922</td>
<td>Two de Broglie papers on massive light quanta.</td>
</tr>
<tr>
<td>1924, 2/2</td>
<td>Gregor Wentzel, „Zur Quantenoptik,“ explains quantum conditions from the interference of all possible (unmechanical) paths of a quantum: Huygens principle for particle trajectories (precursor of Feynman path integral).</td>
</tr>
<tr>
<td>1924, 6/4</td>
<td>Bose sends paper with derivation of Planck law from light quantum statistics to Einstein, published in Z. Phys on 7/2</td>
</tr>
<tr>
<td>1925, 6/19</td>
<td>Alfred Landé, „Lichtquanten und Kohärenz.“ Notes that statistical dependence of Bose statistics can be explained by superposition of quanta.</td>
</tr>
</tbody>
</table>
Transfers

1923, 9-10 Three notes by de Broglie in the Comptes Rendus on matter waves.
1924, 9/20 Einstein, „Quantentheorie des einatomigen idealen Gases“ applies Bose’s statistics to the ideal gas and shows that it delivers Nernst’s law.
1925, 2/9 Einstein, „Quantentheorie des einatomigen idealen Gases, zweite Abhandlung“ Discusses statistical dependence of molecules in Bose’s approach. Finds wave term in fluctuation and refers to de Broglie’s idea of matter waves.
1925, 7/18 Walter Elsasser. “Bemerkungen zur Quantenmechanik freier Elektronen,” interprets electron scattering experiments as showing wave nature of electrons.
1925, 12/15 Schrödinger, „Zur Einsteinschen Gastheorie,“ explains Bose-Einstein statistics as the statistics of quantized matter waves. Discusses particles as wave packets and problem of dispersion. (Unlike de Broglie and Einstein, Schrödinger does not think of two entities, particle and wave.)
1925, 12/22 Kornel Lanczos „Über eine feldmäßige Darstellung der neuen Quantenmechanik.“ Shows that matrix mechanics can be understood as a theory of integral equations and therefore as a continuum theory. (The matrices are represented by continuous kernel functions.) He does not propose a specific kernel, hence also not a specific representation.
1926, 1-6 Schrödinger, „Quantisierung als Eigenwertproblem.”
Questions

- Were there other pathways to wave mechanics?
- What is the role of variational principles for de Broglie and Schrödinger?
- How does the physical interpretation of the Schrödinger wave develop?
  - Charge and current, coupling with the electromagnetic field
- What role plays the development of matrix mechanics for Schrödinger?
- Where do parallels between matrix mechanics and wave mechanics appear and what was the knowledge on which each is based?
  - Hamilton-Jacobi theory?
  - Treatment of dispersion?
  - Hydrogen atom?
Relativity unsolved

What is the role of the relativistic approach in the derivation of the Schrödinger equation?

Famously, Schrödinger first tries to find relativistic wave equation but runs into problems.

Uneasy references:

“It may be objected... This, however, ... is wholly avoided when the relativistic theory is developed and makes a profounder insight possible. ... But unfortunately the correct establishment of the latter meets still with certain difficulties ...” (Quantisation 1)

Similar remarks in 2 and 3.

When he finally writes a section about the relativistic treatment with electromagnetic field in Q4, not much has changed:

“I can only do this at this point for the one electron problem, and only with the gratest possible reserve ...”

Problem of relativity tied up with the problem of treatment of full electromagnetic interaction.
Prospects and Hopes


Shows that matrices can be understood as operators in the space of solutions of Schrödinger equation. Especially interesting is the last section “Comparison of the Two Theories. Prospect of a Classical Understanding of the Intensity and Polarisation of the Emitted Radiation.”

Programmatic statement.

“Perhaps the cardinal question of all atomic dynamics is, as we know, that of the coupling between the dynamic process in the atom and the electromagnetic field.”

Matter field can describe this coupling naturally, without need to quantize the electromagnetic field. Certainly, this is just a prospect at this point, no field equation for particle in electromagnetic field has been given yet.
Prospects and Hopes

Followed by “sketch of the radiation mechanism” assuming that charge density is given by

\[ \text{Re} (\psi \frac{\partial \psi}{\partial t}) \]

Still absolutely preliminary--presumably just staking his claim.

But, the “cardinal question” can also be read as a criterion of adequacy for his own theory. This has a constructive side:

“In any case, it is worth while attempting the representation of the coupling in such a way that we bring into the unchanged Maxwell-Lorentz equations as four-current a four-dimensional vector, which has been suitably derived from the mechanical field scalar \( \psi \) ...”

Heuristic for a full theory?
Not followed up in the last two papers.
The Notebooks

Schrödinger’s notebooks as an obvious resource.
Certainly not unknown, but also not very thoroughly studied: Christian Joas has already identified one additional notebook from early 1926.
New scans from Vienna, with transcriptions of shorthand parts.
Definitely worth a second look.

Quantum History Website