1 Poincaré's Electromagnetic Quantum Mechanics

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The rise of quantum physics is considered by outlining the historical context in which different conceptions of Nature (mechanistic, thermodynamic and electromagnetic ones) were in competition to give a foundation to physics. In particular, the roots of quantum physics within the electromagnetic conception of Nature and Poincaré's quantum electromagnetic mechanics are analysed.

Introduction

Conceptions of Nature

As well known, in the late XIXth century physics was no more mechanics only, but also thermodynamics and electrodynamics. This new situation implied the problem of the very foundations of physics, and the correlated issue of the hierarchical relations among these different physical disciplines.[1] There were at least four different "fighting" conceptions of Nature. The so-called *Energetic* conception of Nature, which was looking at energy as the fundamental unifying concept of physics and had its most important proponents in Georg Helm (1851–1923) and Wilhelm Ostwald (1853–1932).

The *Thermodynamic* conception of Nature, which had energy, entropy and system as fundamental concepts and was looking at thermodynamics as the real foundation block of physics. Its major exponents were Pierre Duhem (1861–1916) and Max Planck (1858–1947).

The *Mechanical* conception of Nature, which was the most conservative one as searching for a mechanical reduction of the other physical disciplines and of all the physical concepts in terms of mass, space and time by means of the models of material point and action at-a-distance forces. Hermann von Helmholtz (1821–1894), Heinrich Hertz (1857–1894) and Ludwig Boltzmann (1844–1906) were the most representative scientists of this perspective.

The *Electromagnetic* conception of Nature, based on the concepts of field, energy and charge was looking at electromagnetism theory as the foundation level of the other physical disciplines. Among the physicists who gave the most relevant contributions to this perspective there are: Hendrik Antoon Lorentz (1853–1928), Joseph Larmor (1857–1942), Wilhelm Wien (1864–1928), Max Abraham (1875–1922) and Henry Poincaré (1854–1912). The electromagnetic conception of Nature has deep roots in the history of

[1] R. McCormmach, C. Jungnickel, *Intellectual Mastery of Nature: Theoretical Physics from Ohm to Einstein*, I–II, The University of Chicago Press, Chicago 1986, II vol., pp. 211–253; E. Giannetto, *Saggi di storie del pensiero scientifico*, Sestante for Bergamo University Press, Bergamo 2005, pp. 299–321.

mankind and certainly has been developed by the elaboration of the Brunian-Leibnizian physics and tradition. On one side, it has been developed within the German physics or *Naturphilosophie*, on the other side mainly within English physics.

William Gilbert (1540–1603) and then the same Johannes Kepler (1571–1630) were thinking about magnetism as the force which rules the order of our cosmos, of our Copernican world, and Athanasius Kircher (1602–1680) developed a theology of magnetism and of the magnetic Divine Universal Love.

Indeed, after the process by which Newtonian gravitation was reduced from a divine active force to a passive property of inertial matter and Newton's theology of gravitation was given up and mechanistic conception of Nature came to dominate, electricity came back to be considered the way to a new vitalistic conception of Nature. Electricity was considered an active force which could have been the origin of animated life, that is an active vital force, the Leibniz' internal *vis viva*, as well as the same *psyché* within things—a sort of electric unconscious—or the same Anima Mundi. Many theologians and physicists, like Prokop Divisch (1698–1765), Friedrich Christoph Oetinger (1702–1782), Johan Ludwig Fricker (1729–1766), Gottlieb Friedrich Rösler (1740–1790), developed a very theology and psychology of electricity. The controversy on animal electricity at the end of XVIII and at the beginning of XIX century between Luigi Galvani (1737–1798) and Alessandro Volta (1745–1827), gave another turn to the consideration of the problem and its resolution with the dominance Volta's perspective and his presentation, in 1800, of the first 'electric machine', the battery, pointed out the victory of the mechanistic view and the reduction of life to mechanisms to which even electricity could have been assimilated. It was the romantic physicist Johan Wilhelm Ritter (1776–1810) who turned Volta's interpretation upside down, stating that, because there was not a specific animal electricity, the whole Nature was a living and animated being just for the presence of electricity. Electric fluid was the *psyché* of everything. Romanticism continued to develop these ideas and Franz Anton Mesmer (1734–1815) spoke about animal magnetism, about a magnetic fluid as a universal soul, about *psyché* as a magnetic nervous fluid, about psychical sickness as magnetic diseases which could be healed by magnetic hypnotism.

Maxwell electromagnetism had shown that physical reality was not only inertial and passive matter, but also dynamical, active electromagnetic field, irreducible to a mechanical matter model. Furthermore, Maxwell equations present vacuum solutions, that is in absence of charged matter: electromagnetic field exists even when there is no matter. Thus, the possibility of a new non-dualistic view of physical reality was considered: if matter cannot exist without electromagnetic field and electromagnetic field can exist without matter, electromagnetic field could be the only physical reality and matter could be derived from the field.

Electromagnetic Conception of Nature and Relativity

Usually, the electromagnetic conception of Nature has been considered as superseded by the developments of XXth century physics. However, a deep historical inquiry shows that the electromagnetic conception of Nature is at the roots of both the relativistic and quantum transformations of physics.

Concerning relativity, the 1900, 1902, 1904 and (5 June) 1905 papers written by

Poincaré[2] show as special relativity dynamics derived from, and was a first realization of, the electromagnetic conception of nature. Einstein's (30 June) 1905 paper was only an incomplete mechanistic version of this new dynamics. This historical recognition is also fundamental to understand the first reception of special relativistic dynamics in all countries, and in particular in Italy.

A first complete presentation of this new dynamics appeared in the July 1905 paper written by Poincaré and published in 1906.[3] In this paper the new dynamics was presented as an invariant one by the Lorentz-Poincaré transformation group, and it was derived by Maxwell's theory of electromagnetism and contained also a theory of gravitation (absent in Einstein's 1905 paper).

The starting point was electromagnetic self-induction phenomenon related to the so-called radiation reaction. When a charged particle is submitted to the action of an electromagnetic field, it is accelerated and it irradiates. This radiation modifies the field and the new field modifies the acceleration of the particle, which again irradiates and so on. In this way, the electromagnetic field depends on all the time derivatives of position up to the infinite one. This means that there is also a contribution to the field force proportional to the acceleration, the coefficient of which involves an electromagnetic mass, that is an electromagnetic contribution to the particle inertia.

At this point, the question was: is it possible that mechanical (inertial and gravitational) mass was not a primitive concept and indeed is wholly due to this electromagnetic effect? Poincaré, among other scientists, realized that this was the case also for non-charged matter as long as is constituted by charged particles: that is mechanical mass was nothing else than electromagnetic mass, and electromagnetic mass is not a static fixed quantity but depends on velocity. Mass is so related to the electromagnetic field energy by the today well-known (now considered from a mechanistic and not electromagnetic perspective) equation: $m = E_{e.m.field}/c^2$.

If mass is nothing else than electromagnetic field energy and charge can be defined, via Gauss' theorem, to the electric field flux through a certain space surface, matter can be completely understood in terms of the electromagnetic field, and it has also active and dynamical features beyond the passive and inertial ones. If mass must be understood in terms of the electromagnetic field, mechanics must be derived by electromagnetism theory which becomes the fundamental theory of physics. If mass changes with velocity, Newtonian mechanics is no more valid and must be modified. The new mechanics must have the same invariance group of electromagnetic theory, that is the Lorentz-Poincaré transformation group, to which a new relativity principle and a new gravitation theory (even gravitational mass changes with velocity) must also be conformed.

From Poincaré's perspective even gravitation is of electromagnetic origin. However, the new gravitational theory developed by Einstein's general relativity theory did not take count of this idea.[4] David Hilbert, simultaneously with Einstein, developed the

[2] H. Poincaré, *La mesure de temps*, in *Revue de Métaphysique et Morale* **6**, 1 (1898); H. Poincaré, *La théorie de Lorentz et le principe de réaction*, Arch. Néerl. **5**, 252 (1900); H. Poincaré, *La Science et l'Hypothèse, Flammarion, Paris 1902; H. Poincaré, L'état actuel et l'avenir de la Physique mathématique*, in *Bulletin des Sciences Mathématiques* **28**, 302 (1904); H. Poincaré, *Sur la dynamique de l'électron*, in *Comptes Rendus de l'Académie des Sciences* **140**, 1504 (1905).

[3] H. Poincaré, *Sur la dynamique de l'électron*, in *Rendiconti del Circolo Matematico di Palermo* **21**, 129 (1906).

[4] A. Einstein, *Die Feldgleichungen der Gravitation*, in *Königlich Preußische Akademie der Wissenschaften*

same gravitational field equations.[5]

The problem of the priority of Einstein or Hilbert, even if historically important, is not the relevant point. Indeed, the fundamental point is that in Hilbert's perspective matter ($T_{\mu\nu}$) is considered as of electromagnetic origin: Hilbert and Einstein equations are mathematically equivalent, but they do not have the same physical meaning. Hilbert's point of view is related to a synthesis of the electromagnetic theory of Gustav Mie (1868–1957)[6] and Einstein theory of gravitation: Hilbert equations give automatically also Maxwell generalized electromagnetic field equations, which follow from the space-time structure induced by "electromagnetic matter."[7]

Thus, it can be traced an evolution line, within the electromagnetic conception of Nature, which started from Poincaré's special-relativistic dynamics and through Mie's theory lead to Hilbert's general-relativistic dynamics. And indeed, by the Hilbert electromagnetic general relativity, that is by the Hilbert electromagnetic theory of matter and gravitation, the cosmic and universal order came back to be related to magnetism as in the first proposals by Gilbert, Kepler and Kircher.

Electromagnetic Conception of Nature and Quantum Physics

The rising of quantum physics is conventionally related to the works of Planck during the years 1899–1900.[8] However, Joseph Larmor, within an electromagnetic conception of Nature, was working to understand the atomic structure of matter in terms of the electromagnetic field at least since 1893.[9] After leaving the idea of a "vortex atom", he considered the electrons as vortices into the sea of the electromagnetic field: this idea lead him to what, many years later, was called a "quantum atom". Electrons as rotations into the electromagnetic field constitute stable, stationary non-radiant configurations of atoms: these configurations correspond to given discrete values of the conserved angular momentum. Radiation is emitted or absorbed by atoms by impulses only when these

(Berlin), Sitzungsberichte, 1915, pp. 844–847.

[5] D. Hilbert, *Die Grundlagen der Physik (Erste Mitteilung)*, in *Nachrichten von der Königlich Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-physikalische Klasse*, Berlin 1916, pp. 395–407.

[6] G. Mie, *Grundlagen einer Theorie der Materie, Erste Mitteilung*, in *Annalen der Physik* **37** (1912) pp. 511–534; *Zweite Mitteilung*, in *Annalen der Physik* **39** (1912) pp. 1–40; *Dritte Mitteilung*, in *Annalen der Physik* **40** (1913) pp. 1–66.

[7] E. R. A. Giannetto, *Einstein, Hilbert and the Origins of the General Relativity Theory*, in press.

[8] M. Jammer, *The Conceptual Development of Quantum Mechanics*, McGraw-Hill, New York 1966, pp. 1–61; M. Planck, *Über irreversible Strahlungsvorgänge*, in *Berliner Berichte*, 18 May 1899, 440 (1899); M. Planck, *Zur Theorie des Gesetzes der Energieverteilung im Normalspektrum*, in *Verhandlungen der Deutschen Pysikalischen Gesellschaft* **2** (14 December), 237 (1900), engl. tr., *On the Theory of the Energy Distribution Law of the Normal Spectrum*, in D. ter Haar, *The Old Quantum Theory*, Pergamon Press, Oxford 1967, pp. 82–90.

[9] J. Larmor, *A Dynamical Theory of the Electric and Luminiferous Medium*, abstract, in *Proc. Roy. Soc.* **54**, 438 (1893); part I, in *Phil. Trans. Roy. Soc.* **185**, 719 (1894); part II abstract, in *Proc. Roy. Soc.* **58**, 222 (1895); part II, in *Phil. Trans. Roy. Soc.* **186**, 695 (1895); part III abstract, in *Proc. Roy. Soc.* **61**, 272 (1897); part III, in *Phil. Trans. Roy. Soc.* **A190**, 205 (1897); J. Larmor, *On the theory of the magnetic influence on spectra; and on the radiation of moving ions*, in *Phil. Mag. (5)* **44**, 503 (1897); J. Larmor, *Aether and Matter*, Cambridge University Press, Cambridge 1900; B. Giusti Doran, *Origins and Consolidation of Field Theory in Nineteenth-Century Britain: From the Mechanical to the Electromagnetic View of Nature*, in *Historical Studies in the Physical Sciences* **6**, (1975), Princeton University Press, Princeton.

configurations change in respect to the minimal total energy. Thus, emission of radiation and loss of energy were not related to the absolute translations of the electron as an accelerated, charged material particle, but to the relative changes (within the atoms) of the inertial rotational motions constituting electrons (in any stable state the change of velocity in a period is zero). This idea furnished an explanation of atomic spectra and even a prediction of the Zeeman effect. This electromagnetic conception of the atomic matter structure, that is the recognition of these atomic matter structures within the electromagnetic field, Larmor understood, would be also the key to the calculus of specific heats in terms of internal energy and equal partition of energy within the kinetic theory of gases.

Planck wanted to show the universality of thermodynamics and its second principle showing that it holds also for electromagnetic phenomena. Planck was forced to use Boltzmann's statistical thermodynamics concept of entropy, but showed that thermodynamics cannot be reduced to mechanics because heat is not only disordered matter motion but also electromagnetic radiation and that thermodynamics could be deduced by electromagnetism theory too. In 1900 Planck introduced discrete values of energy as heuristic tool within statistical thermodynamics of radiation to fit black-body radiation distribution experimental data. That is, energy was treated by Planck not as a continuous mathematical variable, but discrete:

$$E = nh\nu$$

where n is an integral number and so energy is given by an integral multiple of the product of a universal constant $h = 6.5510^{-27}$erg/sec with the physical dimension of an action and the radiation frequency. Planck's words made reference to "energy elements" (*Energieelemente*), but Planck did not want to introduce an essential discontinuity within Nature but only to solve by the mathematical artifact of discreteness the problem to fit experimental data: he did not want to modify classical physics or to make a revolution. In 1899 Planck had already introduced this constant naming it "b" and not "h", it did not denote an action and it was a constant in the different theoretical context of finding an absolute system of natural units of measure.

The first actual physical meaning to this constant was given not by Einstein, but by Larmor in 1902 within his electromagnetic conception of Nature.[10] Following Larmor, Planck's constant was not related to a mathematical artifact but had to be interpreted in terms of the relationship between matter and (ether) electromagnetic field, that is as the ratio between matter energy (given by electromagnetic field energy) and radiation frequency. Planck's constant, for Larmor, was a quantum of the conserved angular momentum to be related to atomic electrons considered as vortices within electromagnetic field.

Larmor proposed also to leave the abstract oscillator model of matter used by Planck and to take count of the actual electromagnetic nature and origin of matter. This implied

[10] J. Larmor, *Theory of Radiation*, in *Encyclopedia Britannica* **8** (vol. XXXII of the complete work), 120 (1902), Black, London. J. Larmor, *On the application of the method of entropy to radiant energy*, in *Reports Brit. Assoc. Adv. Sci. 1902*, 546 (1903) (abstract of a paper presented at the Belfast meeting); J. Larmor, *On the statistical and thermodynamical relations of radiant energy*, in *Proc. Roy. Soc. (London)* **A83**, 82 (1909); J. Larmor, *Preface* (1911) *to The Scientific Papers of S. B. McLaren*, Cambridge University Press, Cambridge 1925.

to use the simple idea of 'elementary receptacles of energy', that is of cells in the phase space of physical systems. This idea was deduced from the consideration of the nature of radiation, constituted by discrete elements given by short trains of simple undulations. The phase space reformulation of Planck's problem lead to the discreteness of the atomic conserved angular momentum from which was deduced the discreteness of energy. J. W. Nicholson in 1912[11] explored this explanation of the atomic structure and his work was the starting point of Niels Bohr's model.

From Larmor's perspective, from the electromagnetic conception of Nature, the discrete, discontinuous, quantum nature of matter and radiation is easily understood because matter is derived from the fundamental physical reality given by the electromagnetic field. Thus, electromagnetic field must present wave but also corpuscular aspects to explain the origin of matter, and matter particles must present corpuscular but also wave aspects as long as they derive from the electromagnetic field.

Bohr[12] reconsidered Nicholson's model but completely changing its meaning: atom was no more understood in terms of the electromagnetic conception of Nature but in terms of an axiomatic approach in which the meaning of Planck's constant is no more given by the electromagnetic nature of the atomic matter structure but by an abstract quantum of mechanical action. Bohr followed Arnold Sommerfeld's perspective[13] which presumed to understand all the things in terms of an a priori assumed and unexplained constant, that is Planck's constant: electromagnetic as well as thermodynamic and mechanical models were considered to be no more suitable because electromagnetic field theory as well as thermodynamics and mechanics must be reformulated in order to fit experiments and to overcome the problem of their incompatibility. However, Sommerfeld and Bohr seem to not understand that their interpretation of Planck's constant was mechanical and this put mechanics at the fundamental level of physics, restating a new mechanistic perspective. It happened something like to the procedure of axiomatization which lead to the loss of electromagnetic meaning to the light velocity constant c in the mechanistic version of relativity dynamics given by Einstein. The meaning variance of a revolutionary item (c as well as h), together with the change in its "title" ("Universal Constant"), is a well known process which leads to a restoration, to a dogma to be understood "mechanically" and to a myth of the foundations of a new religion as well as a new scientific theory.

From Larmor's perspective, Planck's statistical thermodynamics of electromagnetism implied that classical electromagnetism continuous variables lose meaning and cannot be precisely determined, but only probabilistically just in order to derive matter corpuscles from the electromagnetic field.

In 1905–1906 Einstein[14], as well as he had done with Poincaré's new electromagnetic relativistic dynamics, by criticizing Planck noted the discontinuous and probabilistic character of radiation but inverted Larmor's perspective and introduced the quanta of light to reduce electromagnetism (as a statistical theory) to corpuscular mechanics.

[11] J. W. Nicholson, in *Monthly Notices of the Royal Astronomical Society* **72**, 49, 139, 677, 693, 729 (1912).

[12] N. Bohr, in *Philosophical Magazine* **26**, 1, 476, 857 (1913).

[13] A. Sommerfeld, in *Physikalische Zeitschrift* **12**, 1057 (1911).

[14] A. Einstein, *Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt*, in *Annalen der Physik* **17**, 132 (1905); A. Einstein, *Zur Theorie der Lichterzeugung und Lichtabsorption*, in *Annalen der Physik* **20**, 199 (1906).

Poincaré's New Quantum Electromagnetic Mechanics

In 1911 there was the famous first Solvay Conference on the problems raised by Planck's hypothesis and Einstein's quanta. Poincaré was present and participated actively to the debate: here, he understood immediately that physics was at the threshold of the deepest revolution ever happened. It could imply the renounce to the differential equations as (means to formulate) physical laws.[15] In 1911–1912, Poincaré wrote and published two important papers:[16] the first was presented to the *Académie des Sciences* on 4 December 1911. Poincaré showed that Planck's black body law implies necessarily the quanta hypothesis and these new discontinuous characters of light and electromagnetic field cannot be understood in terms of the old corpuscular mechanics, and, on the contrary, these changes within electromagnetic theory imply a new mechanics. Indeed, if mechanics has to be built on electromagnetism and electromagnetism must be changed, then also mechanics must be modified: there must be a new "electromagnetic dynamics".

Poincaré proceeded in this way:[17] let be a system, whose state is defined by n parameters $x_1, x_2, x_3, \dots x_n$. Let be the evolution laws of these parameters formulated by the following differential equations: $\frac{dx_k}{dt} = u_k$.

Let be WdJ the probability that the point representing the system state be in the volume dJ of the x_k-space; then W, the probability density, must satisfy the equation

$$\sum_k \partial \left(\frac{W u_k}{\partial x_k} \right) = 0,$$

where the u_k are the generalized velocities and the equation, as it will be shown, is the same continuity equation that must be satisfied by the Jacobi last multiplier K.[18]

When we deal with classical mechanics indeed we can write the Jacobi equations of

[15]See the *Discussion du rapport de M. Einstein*, in M. P. Langevin et M. de Broglie (eds.), *La théorie du rayonnement et les quanta. Rapports et discussions de la Réunion tenue à Bruxelles, du 30 Octobre au 3 Novembre 1911 sous les auspices de M. E. Solvay*, Gauthier-Villars, Paris 1912, pp. 436–454, in particular p. 451 and *Abhandlungen der deutschen Bunsengesellschaft* **7** , pp. 330–364;

[16]H. Poincaré, *Sur la théorie des quanta*, in *Comptes Rendus de l'Académie des Sciences*, v. **153** (1912), pp. 1103–1108, reprinted in H. Poincaré, *Œuvres de Henri Poincaré*, I–XI, Gauthier-Villars, Paris 1934–1956, v. **IX**, pp. 620–625; *Sur la théorie des quanta*, in *Journal de Physique théorique et appliquée*, v. **2** (1912), pp. 5–34, reprinted in*Œuvres*, v. **IX**, op. cit., pp. 626–653; *L'hypothèse des quanta*, in *Revue Scientifique*, v. **50** (1912), pp. 225–232, reprinted in *Œuvres*, v. **IX**, op. cit., pp. 654–668 and as chapter 6 in H. Poincaré, *Dernières pensées*, Flammarion, Paris 1913; H. Poincaré, *Les rapports de la matière et l'éther*, in *Journal de physique théorique et appliquée*, ser 5, **2** (1912), pp. 347–360, reprinted in *Œuvres*, v. **IX**, op. cit., pp. 669–682 and as chapter 7 in H. Poincaré, *Dernières pensées*, op. cit.. See also: H. Poincaré, *L'évolution des lois*, conference delivered at the Congresso di Filosofia di Bologna on 8 April 1911, in *Scientia*, v. **IX** (1911), pp. 275–292, reprinted as chapter 1 in *Dernières pensées*, op. cit.

[17]See footnote 16.

[18]C. G. J. Jacobi, in *Crelle's Journal* **XXVII** (1844) p. 199 and **XXIX** p. 213, 388; A. R. Forsyth, *A Treatise on Differential Equations*, MacMillan, London 1885, sixth edition 1948, pp. 356–366; E. Whittaker, *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies*, Cambridge University Press, Cambridge 1904, fourth edition 1960, pp. 267–287; R. H. Fowler, *Statistical Mechanics—The Theory of the Properties of Matter in Equilibrium*, Cambridge University Press, Cambridge 1929, second edition 1936, reprinted in 1955, pp. 11–15; D. Buoccaletti, G. Pucacco, *Theory of Orbits, vol. I, Integrable Systems and Non-perturbative Methods*, Springer Verlag, Berlin 1996, pp. 61–72.

motion in the canonical form:

$$\frac{dq_i}{dt} = \frac{\partial T}{\partial p_i} \qquad \text{and} \qquad \frac{dp_i}{dt} = -\frac{\partial T}{\partial q_i} + Q_i$$

where

$$Q_i = \sum_i (X_k \frac{\partial x_k}{\partial q_i} + Y_k \frac{\partial y_k}{\partial q_i} + Z_k \frac{\partial z_k}{\partial q_i})$$

are generalized forces.

These equations are more general than Hamilton's ones, because they do not presuppose the existence of a potential function.

The Jacobi last multiplier is so defined:

$$0 = \frac{d\ (\log K)}{dt} + \sum_k \partial \frac{(\frac{\partial T}{\partial p_k})}{\partial q_k} + \sum_k \partial \frac{(\frac{\partial p_k}{\partial dt})}{\partial q_k}$$

From this equation it follows:

$$\frac{1}{K}\frac{dK}{dt} + \sum_k \partial \frac{(\frac{dq_k}{dt})}{\partial q_k} + \sum_k \partial \frac{(-\frac{\partial T}{\partial q_k} + Q_k)}{\partial p_k} = 0$$

For K different from zero, it yields:

$$\frac{dK}{dt} + K \sum_k \partial \frac{(\frac{dq_k}{dt})}{\partial q_k} + \sum_k \partial \frac{(-\frac{\partial T}{\partial q_k} + Q_k)}{\partial p_k} = 0$$

and so in the other coordinates:

$$\frac{dK}{dt} + K \sum_k \frac{\partial u_k}{\partial x_k} = 0$$

Then, it can be written:

$$\frac{dK}{dt} + K\ \text{div}\ \vec{u} = 0$$

and so:

$$\frac{\partial K}{\partial t} + (\text{div}\ K)\ \vec{u} + K\ \text{div}\ \vec{u} = 0$$

And finally the following continuity equation is obtained:

$$\frac{\partial K}{\partial t} + \text{div}(K\vec{u}) = 0$$

If Hamilton's equations

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i} \qquad \text{and} \qquad \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}$$

hold, then $\frac{\partial Q_k}{\partial p_k} = 0$ and

$$\frac{dq_i}{dt} = \frac{\partial T}{\partial p_i} \qquad \text{and} \qquad \frac{dp_i}{dt} = -\frac{\partial T}{\partial q_i}$$

Thus, it can be deduced that:

$$\sum_k \partial \frac{u_k}{\partial x_k} = \sum_k \partial \frac{\left(\frac{\partial T}{\partial p_k}\right)}{\partial q_k} - \sum_k \partial \frac{\left(\frac{\partial T}{\partial q_k}\right)}{\partial p_k} = 0$$

If $\operatorname{div}(K\vec{u}) = 0$, that is the current is stationary, then

$$(\operatorname{div}\ K)\vec{u} + K\ \operatorname{div}\ \vec{u} = 0$$

and so from

$$\operatorname{div}\ \vec{u} = 0$$

it follows

$$(\operatorname{div}\ K)\vec{u} = 0$$

Therefore, for K different from $K = K(t)$, that is for K independent from time, it yields:

$$\frac{dK}{dt} = \operatorname{div}(K\vec{u}) = 0,$$

and so

$$\frac{dK}{dt} = K\ \operatorname{div}\ \vec{u} + (\operatorname{div}\ K)\vec{u} = 0,$$

and finally

$$\frac{dK}{dt} = (\operatorname{div}\ K)\vec{u} = 0,$$

so that K is independent even from the x_k , that is K is a constant.
Thus, we can choice $K = 1$ and then it is obtained

$$\operatorname{div}(K\vec{u}) = K\ \operatorname{div}\ \vec{u} = 0$$

and so

$$\operatorname{div}\ \vec{u} = 0$$

with Hamilton's equations satisfied. Otherwise, if K is not constant and in general it depends from t and from the x_k , it is

$$\sum_k \partial \left(\frac{K u_k}{\partial x_k}\right) = 0$$

that is the same equation that holds for the probability density W, and so $W = K$: probability density is the Jacobi last multiplier. The condition $W = K = 1$, as Dugas has remarked[19], corresponds to the complete homogeneity of the possibility that the system state representative point is everywhere in the phase space of the q_k and the p_k.

[19] R. Dugas, *Histoire de la mécanique*, Griffon, Neuchâtel 1955, English tr. by J. R. Maddox, *A History of Mechanics*, Dover, New York 1988, pp. 552–553 and 622–626. For other comments to Poincaré's papers, see: M. Planck, *Henri Poincaré und die Quantentheorie*, in *Acta Mathematica* 1, **38** (1921) pp. 387–397; H. A. Lorentz, *Deux Mémoires de Henri Poincaré sur la Physique mathématique*, in H. Poincaré, *Oeuvres de Henri Poincaré*, op. cit., **11**, pp. 247–261; P. Langevin, *L'oeuvre d'Henri Poincaré. Le physicien, in Revue de Métaphysique et de morale*, Supplément au n. 5 (1913), pp. 675–718; R. McCormmach, *Henri Poincaré and the quantum theory*, in *Isis* **58** (1967), pp. 37–55.

Thus, Poincaré proposed to introduce a new Jacobi last multiplier, that is a new probability density in phase space, different from unity and given by an essential discontinuous function just to obtain Planck's law and not Rayleigh-Jeans equation of equipartition.

W must be a function containing factors $w = w(e_k)$ which are zero for values of energy e_k different from a multiple of the quantum e. This property is introduced to give a finite energy electromagnetic radiation: thus, mechanics must be modified to take count of electromagnetic variables, that is of the modified electrodynamics which Planck's law requires. This can be realized trivially by assuming that all the mechanical forces, and so all the forms of exchange of energy, are of electromagnetic nature. The new mechanics is a new quantum electromagnetic mechanics.

This new quantum electromagnetic mechanics, as Poincaré conceived it, is a theory for an isolated system, and rigorously only for the whole universe: the evolution dynamics of the universe thus results to be discontinuous and the universe would jump discontinuously from a state to another one. This implies that it is not possible to distinguish a continuous range of intermediate states and instants too in which no change is present in the universe, and so it yields a discontinuous time: atoms of time must be introduced. This was the first time a *quantum time* was introduced.

If W is an essentially discontinuous function all the equations involving it must be modified by replacing integrals with sums and derivatives and differentials with finite variations, which correspond to quantum discontinuous jumps.

Thus, it yields a finite variation equation for W:

$$\sum_k \frac{\Delta(W(\frac{\Delta x_k}{\Delta t}))}{\Delta x_k} = 0$$

This is an equation for a discontinuous quantum "density matrix" in the phase space or in the action-angle space. The discontinuity of the W function corresponds to the impossibility of simultaneously determining the separate probability distributions of coordinates and momenta as continuous variables, in such a way that a minimum size for phase space cell exists and it is given by

$$\Delta q \; \Delta p = h$$

This is the finite difference relation which must replace the integral equation for an elementary phase space cell introduced by Planck and quoted by Poincaré: this relation implies the mutual dependent variability of coordinates and momenta, which furthermore must vary in jumps.

Poincaré's new equations of motion are the first form of new quantum mechanical equations and can be compared to the successive most general form of quantum mechanical Liouville equations for density matrix[20] when it is impossible to define a Schrödinger wave function: however, Poincaré's equations represent a more radical shift from classical mechanics, because are finite variation equations. From Poincaré perspective, continuity cannot be saved even writing an equation for the probability density, because this must be an essentially discontinuous function. Poincaré's equations are more general than

[20] P. Carruthers and M. M. Nieto, *Phase and Angle Variables in Quantum mechanics*, in *Reviews of Modern Physics* **40** (1968) p. 411.

Schrödinger's ones in a further respect: they do not presuppose the possibility to define a potential function for the interaction, because are derived from Jacobi equations.

Dugas[21] has shown (even by neglecting the potential problem) that in Schrödinger's quantum mechanics the Jacobi last multiplier is given by $\psi^*\psi$ defined in configuration space and in Dirac's spinorial quantum relativistic mechanics by $\psi_k^*\psi_k$ as continuous functions. However, following Poincaré, even probability density functions are discontinuous and physical laws can no more be represented by differential equations.

From this perspective, electromagnetism cannot be reduced to mechanics, but, on the contrary, mechanics must be modified again and in more radical way than by the relativistic electromagnetic dynamics: mechanics must be intrinsically probabilistic even for only one material particle, because the origin of matter is electromagnetic and electromagnetic radiation is discontinuous.

Poincaré's new electromagnetic discontinuous mechanics based on a discontinuous electromagnetic action was mathematically very difficult for the other physicists (Jacobi last multiplier technique was used in celestial mechanics) and was not understood at all: thus, this first form of a new revolutionary electromagnetic quantum mechanics was not accepted.

Concluding Remarks

Only after many years, in 1925, Heisenberg[22] stated the necessity of, and posed the basis for, a new quantum mechanics: his starting point was not the electromagnetic conception of Nature, but an operational perspective. Heisenberg showed that at the atomic or microphysical level the only measurable variables were the electromagnetic variables of frequency and intensity of electromagnetic radiation absorbed or emitted by electrons within atoms. From this point of view, mechanical variables, as long as they are not directly measurable and cannot be objects of absolute experimentation, intuition or visualization at the atomic microphysical level, must be redefined in terms of such measurable electromagnetic variables. This implied, as then stated in 1927 by Heisenberg himself[23], a fundamental indeterminacy of mechanical variables. If physical reality is only what can be experimentally measured, from Heisenberg's perspective the electromagnetic conception of Nature can be deduced without any aprioristic assumption. Its deduction follows merely from the request of an operational definition of physical variables at the microscopic level.

Unfortunately, this original derivation and foundation of quantum mechanics has been completely forgotten and removed. It was for ideological reasons that mechanics must be maintained independent from electromagnetism and at the foundation level of the physical sciences. This priority of mechanics is related to the mechanistic conception of Nature. Considering Nature and the other non-human living beings as machines, that is as inert and passive matter, is the pre-condition to avoid any ethical problem in respect

[21] See footnote 19

[22] W. Heisenberg, *Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen*, in *Zeitschrift für Physik* **33**, 879 (1925); M. Born, W. Heisenberg and P. Jordan, *Zur Quantenmechanik II*, in *Zeitschrift für Physik* **35**, 557 (1926).

[23] W. Heisenberg, *Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik*, in *Zeitschrift für Physik* **43**, 172 (1927).

of Nature and the other non-human living beings and to the complete violent dominion over, and exploitation of, Nature and the other living beings.

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