(Never) mind your p's and q's: Von Neumann versus Jordan on the Foundations of Quantum Theory

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Abstract:

In two papers in 1927, Jordan published his version of a general formalism for quantum mechanics now known as Dirac-Jordan transformation theory. Guided by classical mechanics, Jordan gave transformations from one set of canonical variables q and their conjugate momenta p to other such sets a central role in his formalism. Such transformations are not always unitary, leading to non-Hermitian p's and q's. So wedded was Jordan to classical mechanics that he initially tried to make room in his quantum formalism for such non-Hermitian p's and q's. Moreover, recognizing that for quantities with fully discrete spectra, such as spin, it is impossible to define canonically conjugate variables as q's and p's that satisfy the standard commutation relation $pq - qp = h/2\pi i$, he relaxed the definition of what it means for variables to be canonically conjugate so that such cases could be handled in his formalism as well.

Partly in response to the first of Jordan's two papers, von Neumann, in a series of papers of 1927 that form the basis for his famous 1932 book, developed the modern Hilbert space formalism of quantum mechanics. Jordan objected that von Neumann did not show much interest in canonically conjugate variables or canonical transformations. Von Neumann's view of the appropriate formulation of problems in the new quantum mechanics was very different from Jordan's. Whereas for Jordan, unable to let go of the connection to classical mechanics, the solution of such problems necessarily required the identification of a set of canonically conjugate variables, von Neumann, not constrained by this analogy to classical physics, realized that the identification of a maximal set of commuting operators with simultaneous eigenstates is all that matters. In our talk, we will highlight the elements in the relevant papers of Jordan and von Neumann that bring out this gradual loosening of the ties between the new quantum formalism and classical mechanics.