The Transformation of Mechanics

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QUANTUM HISTORY PROJECT

Outline

- **Revolution or Transformation?** The fate of the knowledge of classical physics.
- **Dizygotic twins?** The quantum revolution and the two versions of the new mechanics.
- **Classical Roots?** The refinement of the correspondence principle vs. the optical-mechanical analogy.

Part I:

Revolution or Transformation?

Challenges to the mechanical worldview

19th century physics:

- **Mechanics** (Newton, Lagrange, Hamilton)
- **Electrodynamics** (Maxwell, Hertz)
- **Thermodynamics** (Helmholtz, Clausius, Gibbs, Nernst, Boltzmann, Planck)

Solvay 1911

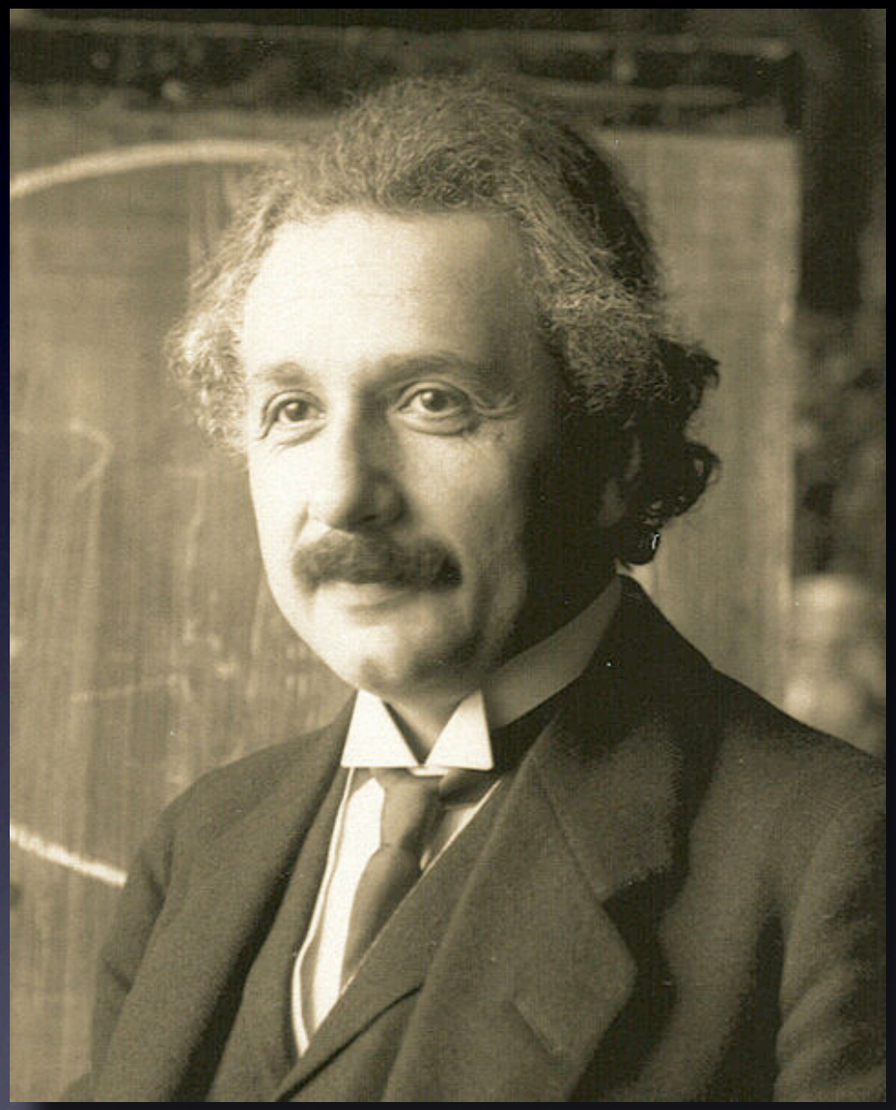
Challenges to the mechanical worldview arise at the borderline between these theories!

Revolution or Transformation?

- Three major **new conceptual frameworks** emerge at the beginning of the 20th century:
  - quantum physics
  - relativity physics
  - statistical physics
- Where did the **knowledge** come from that enabled the development of these frameworks?
- Which role did **previously established knowledge** play?

The Relativity Revolution

- The **paradox of missing knowledge**: Few empirical hints towards a theory radically different from Newton's mechanics.
- Historical research has shown: Relativity theory was a **transformation of classical physics** resulting from a **reorganization** of established knowledge under new principles.
- For example: Re-interpreting inertial forces as the effects of a generalized gravito-inertial field (**Equivalence Principle**).



Albert Einstein (1879–1955)

The Origins of the Quantum Revolution

Mechanics

Conflicts with new empirical evidence:

black-body radiation
atomic spectra
specific heat
X-ray absorption
Stern-Gerlach experiment

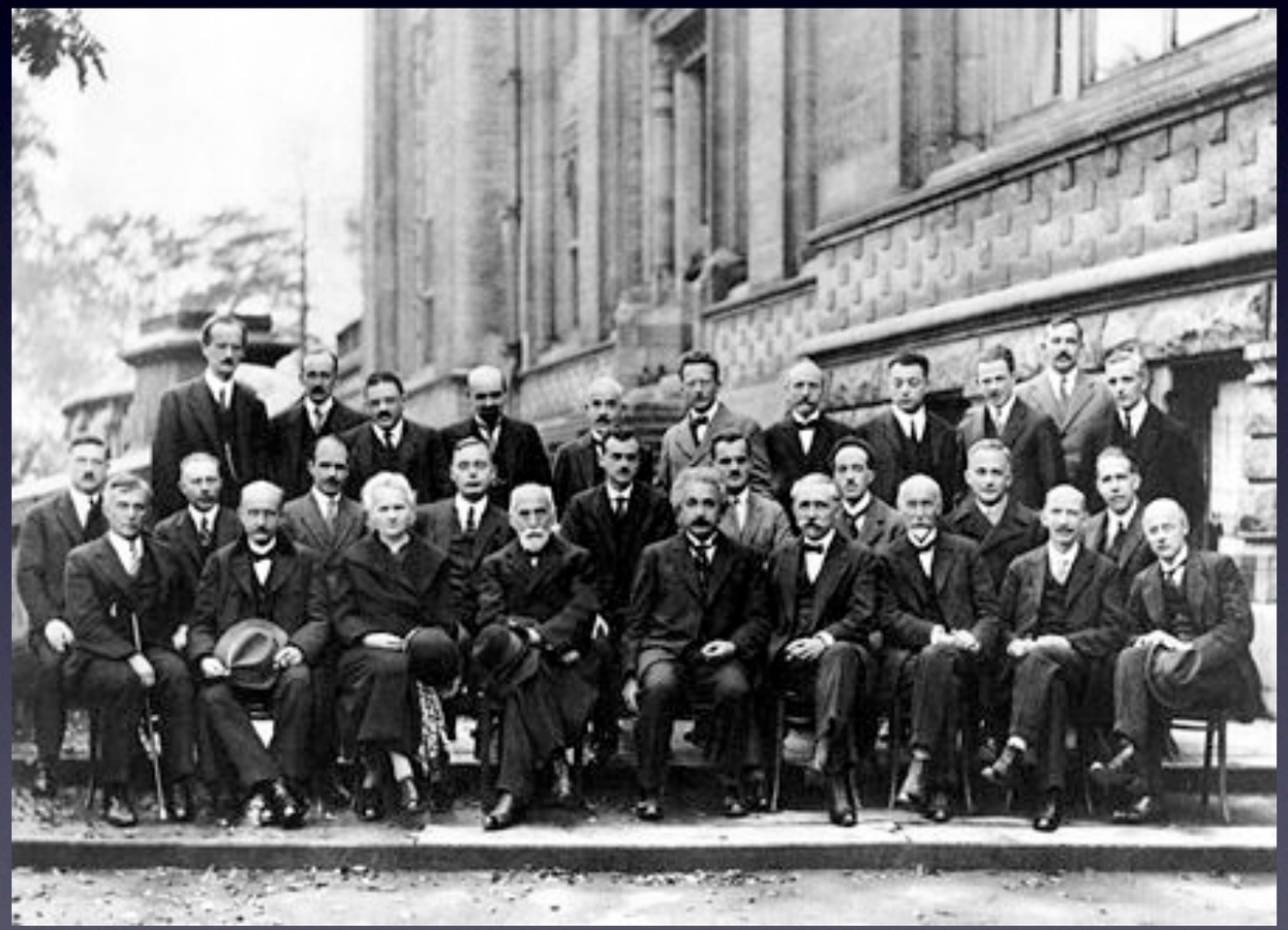
Borderline problems with:

electrodynamics
thermodynamics
chemistry

Quantum Revolution

Quantum vs. Relativity Revolution

- Few **actors** in relativity vs. many in quantum.
- Scarce **empirical basis** in relativity vs. a bulk of new empirical findings in quantum.
- One final **formulation** in relativity vs. two distinct formulations in quantum: matrix and wave mechanics.



Solvay 1927

Old Quantum Theory

- The old quantum theory consisted in augmenting **Hamiltonian mechanics** by auxiliary conditions.
- **Quantum condition:** The action integral around a classical orbit must be an integer multiple of Planck's quantum of action:

$$\oint p dq = nh$$

- **Correspondence principle:** The classical theory of electrodynamics offers a limit which restricts possible transitions between orbits.
- These were **heuristic schemes** rather than full-fledged theory.
- What were the crucial steps in the transition from old quantum theory to either matrix or wave mechanics?

The Crisis of the Old Quantum Theory

- The old quantum theory **failed** to explain many empirical findings: Helium spectrum, Zeeman effect, multiplet structure of atomic spectra, aperiodic phenomena in general.
- From ca. 1923, **doubts** in the validity of the scheme of old quantum theory arose.
- Instead of a heuristic scheme, physicists now sought for a "sharpened" formulation of the **correspondence principle** that would yield a full theory with the explanatory power to tackle the open problems.
- Heisenberg's 1925 **matrix mechanics** was an attempt to accomplish this using insights from the problems that troubled the old quantum theory (e.g., dispersion, multiplet structure).
- In 1926, Schrödinger's **wave mechanics**, however, offered an equally general theory, based on rather different evidence and principles.
- Very rapidly, it became clear that the two new theories are essentially **equivalent**.
- **How can this be?**

Part II:

Dizygotic Twins?

Two New Versions of Mechanics

- Which knowledge enabled the crucial step to the two new versions of mechanics?
- How could there be two distinct approaches to what later turned out to be equivalent in important respects?
- Why was the reformulation of Bohr's correspondence principle crucial for one theory and immaterial for the other?

Candidates for Knowledge Fueling the Crucial Step towards Quantum Mechanics

- 1900 Planck's radiation formula for heat radiation with the help of the energy-frequency relationship
- 1905 Einstein's explanation of the photoelectric effect with the help of the light quantum hypothesis
- 1913 Bohr's explanation of the hydrogen spectrum with the help of his atomic model
- 1916 Schwarzschild's and Epstein's explanation of the Stark effect with the help of a modified Hamiltonian mechanics
- 1916 Einstein's derivation of the black-body radiation formula from the Bohr model with the help of emission and absorption coefficients
- 1923 de Broglie's explanation of Bohr's quantum conditions using a wave theory of matter
- 1924 Kramers' and Heisenberg's explanation of optical dispersion with the help of the correspondence principle
- 1924 Einstein's and Bose's explanation of Nernst's heat theorem with the help of a new statistics

Knowledge Fueling the Crucial Step towards Matrix Mechanics

- 1900 Planck's radiation formula for heat radiation with the help of the energy-frequency relationship
- 1905 Einstein's explanation of the photoelectric effect with the help of the light quantum hypothesis
- 1913 Bohr's explanation of the hydrogen spectrum with the help of his atomic model
- 1916 Schwarzschild's and Epstein's explanation of the Stark effect with the help of a modified Hamiltonian mechanics
- 1916 Einstein's derivation of the black-body radiation formula from the Bohr model with the help of emission and absorption coefficients
- 1923 de Broglie's explanation of Bohr's quantum conditions using a wave theory of matter
- 1924 Kramers' and Heisenberg's explanation of optical dispersion with the help of the correspondence principle
- 1924 Einstein's and Bose's explanation of Nernst's heat theorem with the help of a new statistics

Knowledge Fueling the Crucial Step towards Wave Mechanics

- 1900 Planck's radiation formula for heat radiation with the help of the energy-frequency relationship
- 1905 Einstein's explanation of the photoelectric effect with the help of the light quantum hypothesis
- 1913 Bohr's explanation of the hydrogen spectrum with the help of his atomic model
- 1916 Schwarzschild's and Epstein's explanation of the Stark effect with the help of a modified Hamiltonian mechanics
- 1916 Einstein's derivation of the black-body radiation formula from the Bohr model with the help of emission and absorption coefficients
- 1923 de Broglie's explanation of Bohr's quantum conditions using a wave theory of matter
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Distinct Knowledge Resources for Matrix and Wave Mechanics?

- Crossover Phenomenon:
  - Wave mechanics grew out of attempts to explain the **hydrogen spectrum** and covered **optical dispersion** only in the aftermath.
  - Matrix mechanics grew out of attempts to explain **optical dispersion** dispersion and covered the **hydrogen spectrum** only in the aftermath.
- **How could wave mechanics come ultimately to the same conclusions as matrix mechanics without dispersion theory as an ingredient?**

Pre-established Harmony: Possible reasons?

- Was wave mechanics just a **re-dressing** of matrix mechanics which already was known to Schrödinger?
- Were both theories **incomplete** and did only their synthesis give rise to what we today know as quantum mechanics?
- Does reality enforce **convergence** of different theoretical approaches?
- Were pre-existing **mathematical structures**, such as the Hilbert space formalism, uncovered independently by the two approaches?

Part III:

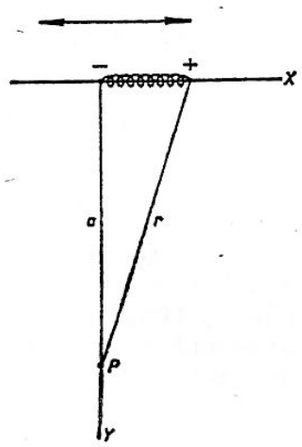
Classical Roots

The Search for a “Sharpening” of the Correspondence Principle

- Around 1924, attempts were made to **“sharpen” the correspondence principle** into a general translation procedure allowing to derive quantum states from a classical description of physical systems.
- e.g., Born’s discretization of differential equations in his 1924 article “Über Quantenmechanik.”
- The successful application of virtual oscillators in the context of dispersion served as a hint that they might be a **model base different from classical orbits** for such a sharpened correspondence principle.

Heisenberg 1925: Umdeutung

Der Grundgedanke ist: In der klassischen Theorie genügt die Kenntnis der Fourierreihe der Bewegung um *alles* auszurechnen, nicht etwa nur



das Dipolmoment (und die Ausstrahlung), sondern auch das Quadrupolmoment, höhere Pole u.s.w. Um ein Beispiel zu geben: Ein anharmonischer Oszillator schwinge in der x-Richtung,

$$x = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \cdots;$$

dann kann man z.B. die periodische Kraft auf einen Punkt P (im Abstande a vom Nullpunkt) ausrechnen und findet

$$K = -\frac{e^2}{a^2} + \frac{e^2}{a^2+x^2} = \frac{e^2}{a^2}\left(-1 + \frac{1}{1+x^2/a^2}\right).$$

Die Fourierentwicklung von $1/(1+x^2/a^2)$ sei

$$b_0 + b_1 \cos \omega t + b_2 \cos 2\omega t + \cdots,$$

dann findet man

$$(1) \quad b_0 = 1 - \frac{a_0^2 + \frac{1}{2}a_1^2 + \cdots}{a_2} + \frac{\cdots}{a^4}; \quad b_1 = -\frac{2(a_0 a_1 + \frac{1}{2}a_1 a_2 + \cdots)}{a^2}; \quad b_2 = \ldots$$

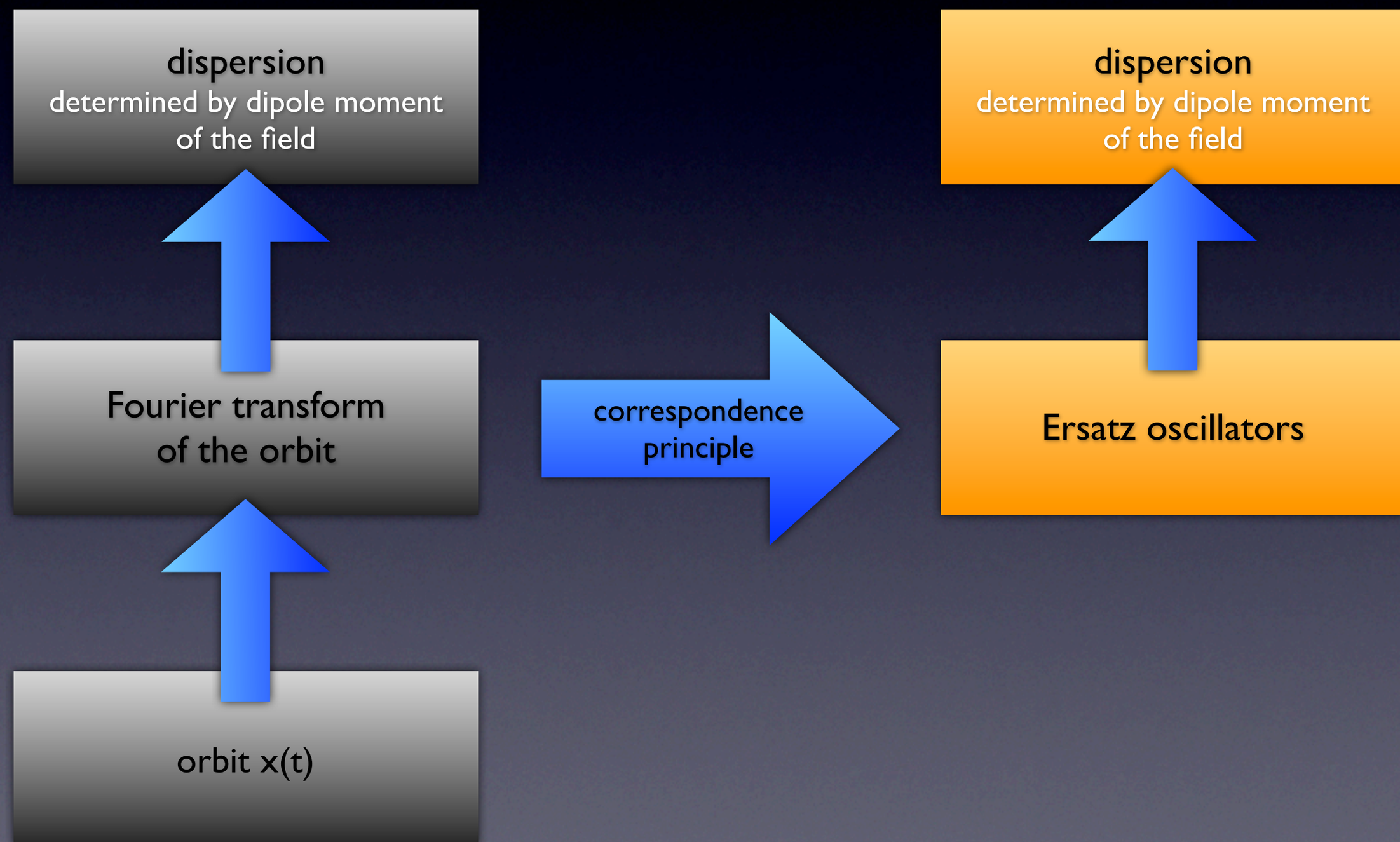
Also die Fourierkoeffizienten sind durch die ursprünglichen a_n ausdrückbar. Es liegt nun nahe, anzunehmen, dass auch in der Quantentheorie

The basic idea is: In the classical theory, knowing the Fourier expansion of the motion is enough to calculate **everything**, not just the dipole moment (and the emission), but also the quadrupole and higher moments, etc.

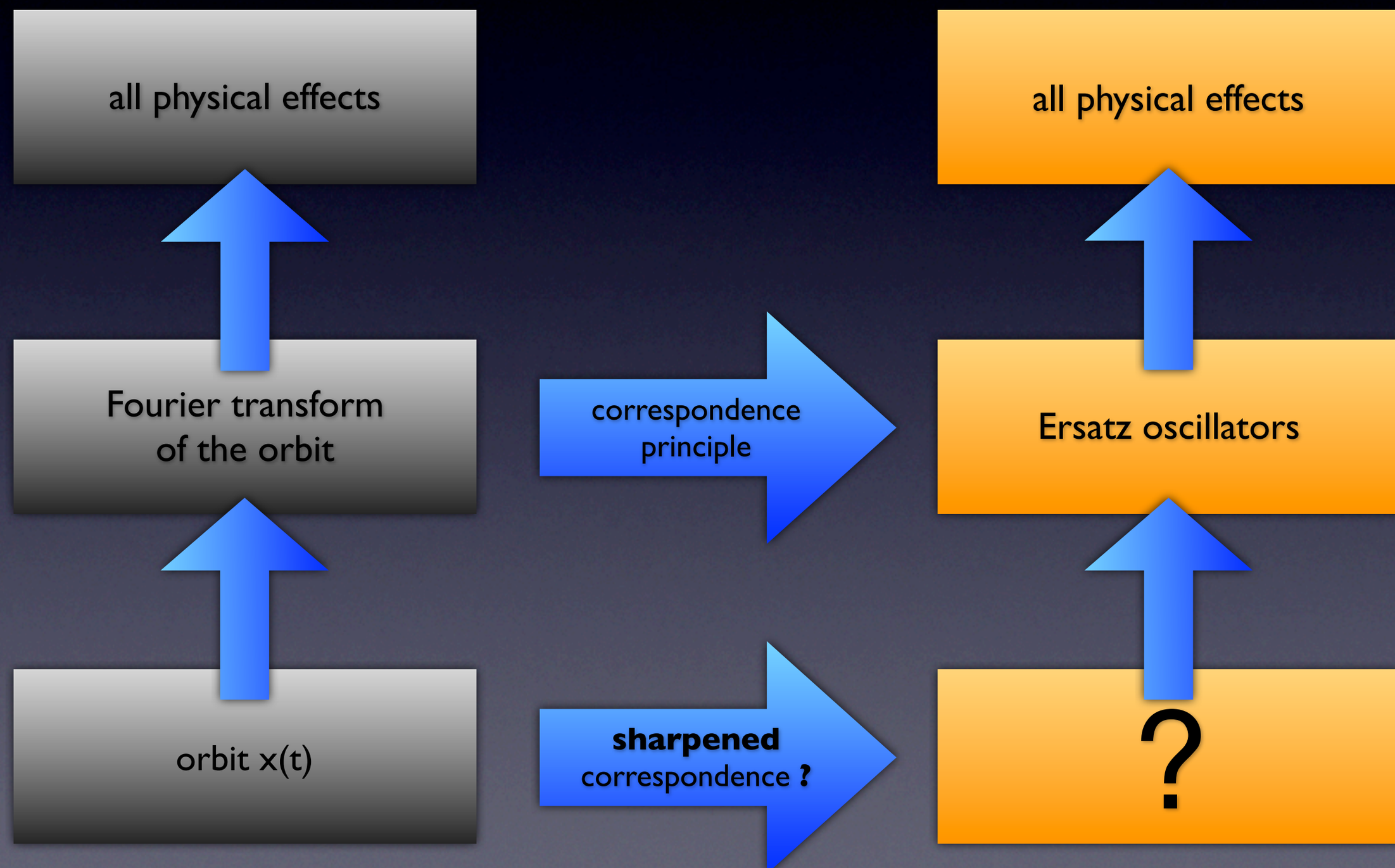
Heisenberg to Kronig, May 1925

Hamiltonian Mechanics → Correspondence Principle → Matrix Mechanics

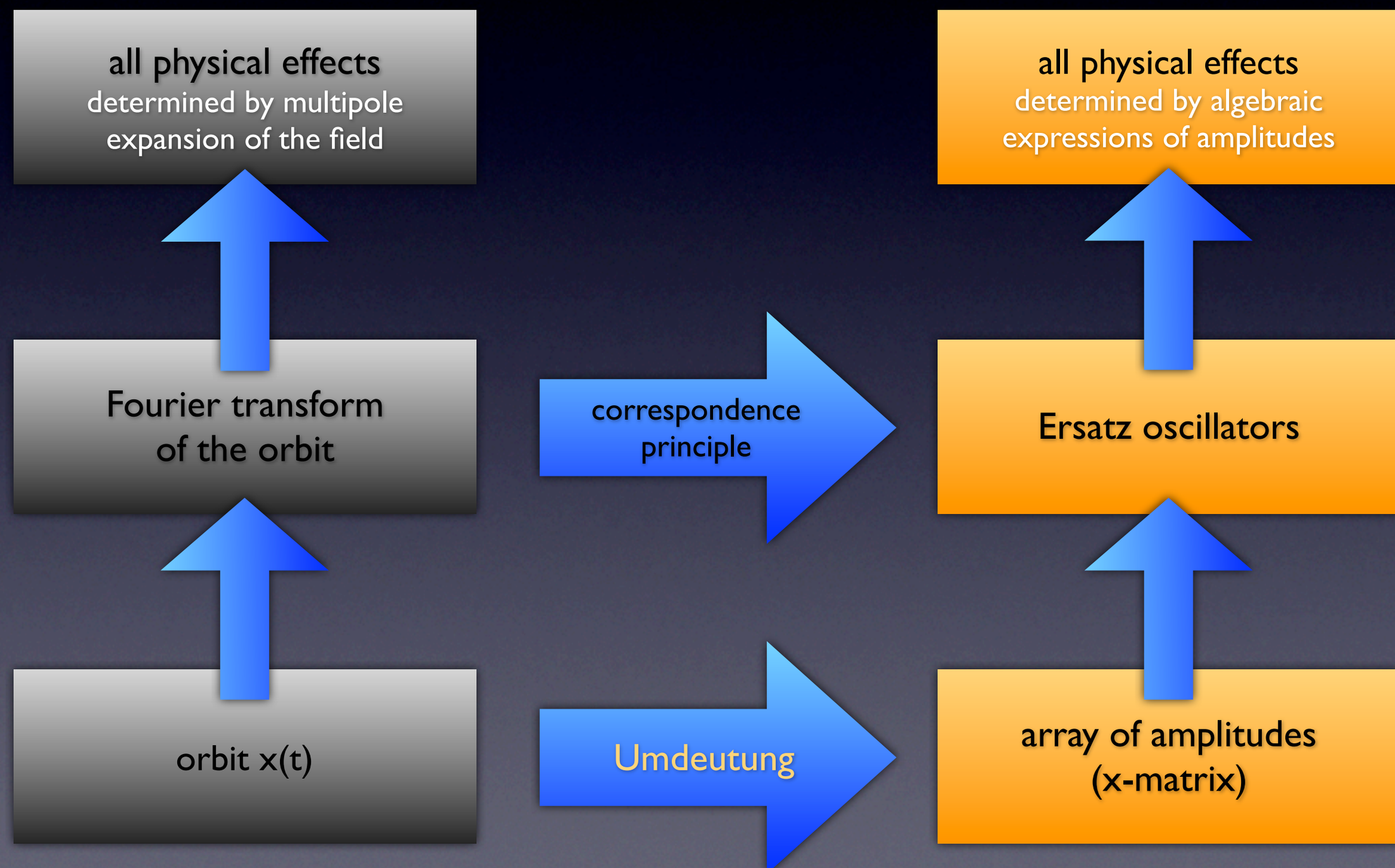
Heisenberg, Kramers (Jan. 1925): Dispersion Theory



The Search for the Sharpened Correspondence Principle



Heisenberg (July 1925): Umdeutung



Heisenberg's Re-Casting of the Correspondence Principle

Die Antwort lautet klassisch offenbar so:

$$\mathfrak{B}_\beta(n)\, e^{i\omega(n)\beta t} = \sum_{-\infty}^{+\infty}{}_\alpha\, \mathfrak{A}_\alpha \mathfrak{A}_{\beta-\alpha}\, e^{i\omega(n)(\alpha+\beta-\alpha)t} \tag{3}$$

bzw.

$$= \int_{-\infty}^{+\infty} \mathfrak{A}_\alpha \mathfrak{A}_{\beta-\alpha}\, e^{i\omega(n)(\alpha+\beta-\alpha)t}\, d\alpha, \tag{4}$$

Quantentheoretisch scheint es die einfachste und natürlichste Annahme, die Beziehungen (3, 4) durch die folgenden zu ersetzen:

$$\mathfrak{B}(n, n-\beta)\, e^{i\omega(n,n-\beta)t} = \sum_{-\infty}^{+\infty}{}_\alpha\, \mathfrak{A}(n, n-\alpha)\, \mathfrak{A}(n-\alpha, n-\beta)\, e^{i\omega(n,n-\beta)t} \tag{7}$$

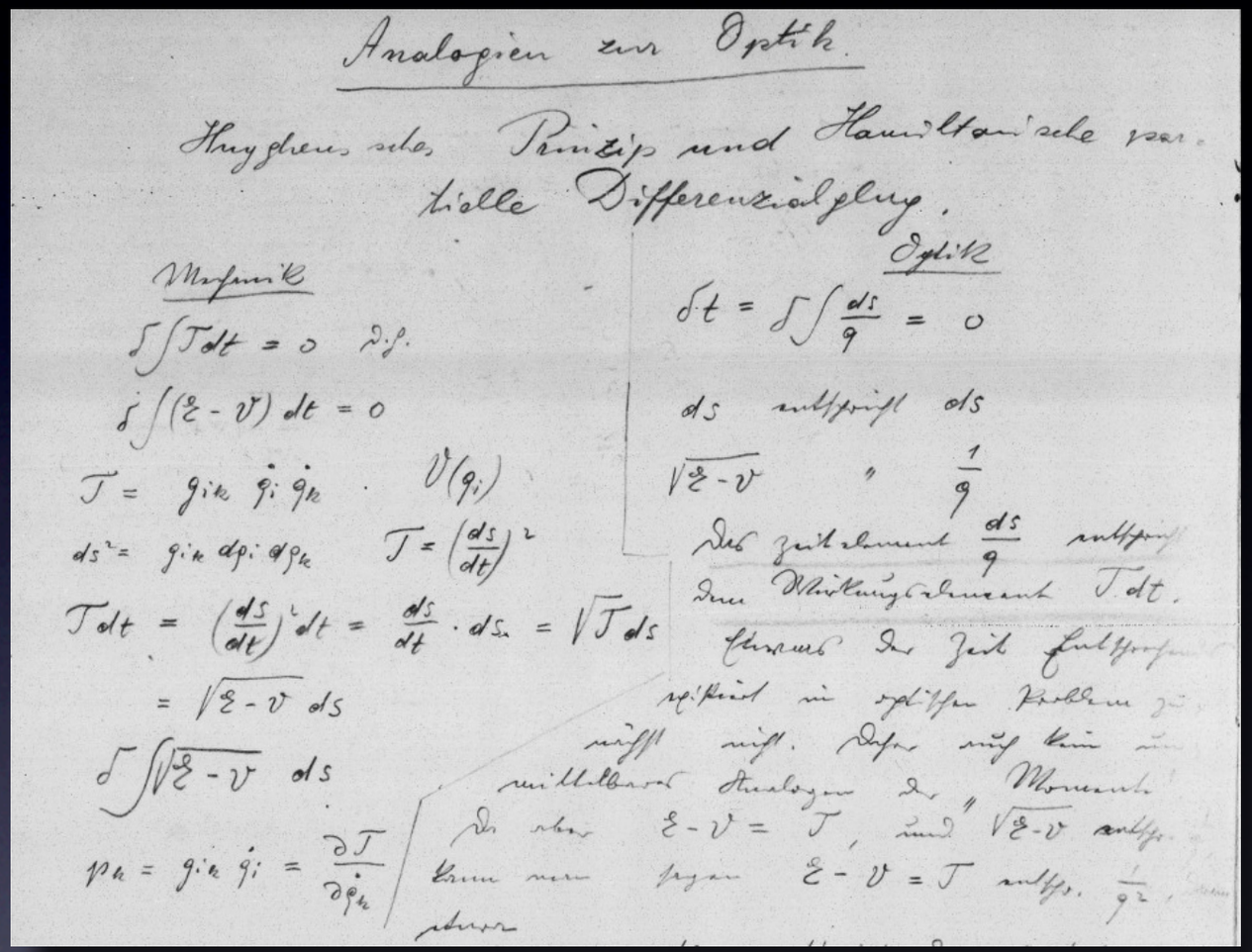
bzw.

$$= \int_{-\infty}^{+\infty} d\alpha\, \mathfrak{A}(n, n-\alpha)\, \mathfrak{A}(n-\alpha, n-\beta)\, e^{i\omega(n,n-\beta)t}; \tag{8}$$

und zwar ergibt sich diese Art der Zusammensetzung nahezu zwangläufig aus der Kombinationsrelation der Frequenzen. Macht man diese An-

Heisenberg, Umdeutung (1925)

Schrödinger 1926: Wave Mechanics

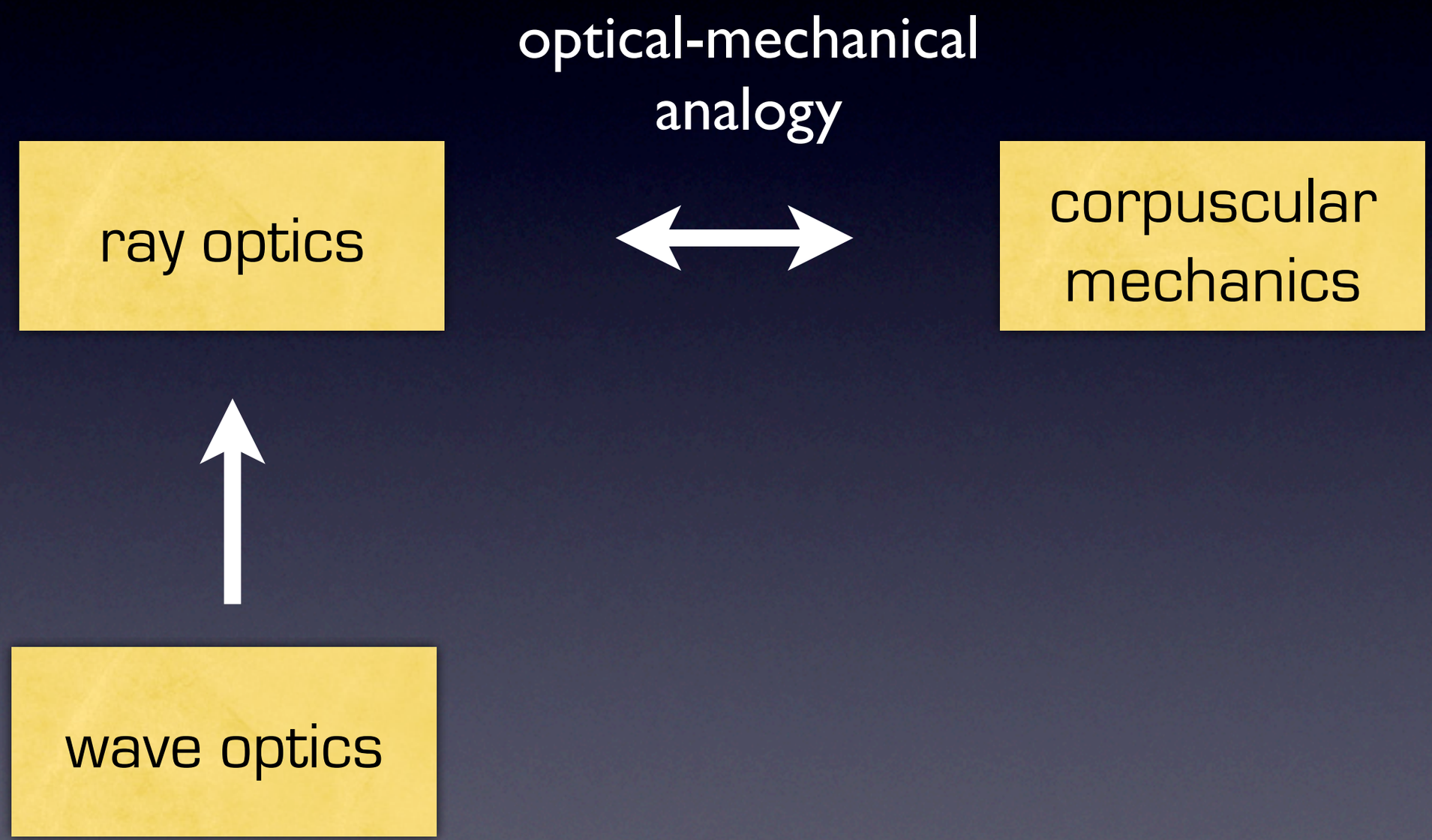


The Hamiltonian analogy in Schrödinger's notebook (ca. 1918–1920).

- Schrödinger found a „wave“ generalization of Hamiltonian mechanics through the optical-mechanical analogy.
- This led him to his new mechanics.
- This also explains Schrödinger's later stance on interpretation.

Hamiltonian Mechanics → Optical-Mechanical Analogy → Wave Mechanics

Hamilton's Optical-Mechanical Analogy



abstract attempt at unifying optics and mechanics

Schrödinger's Completion of Hamilton's Analogy

optical-mechanical analogy

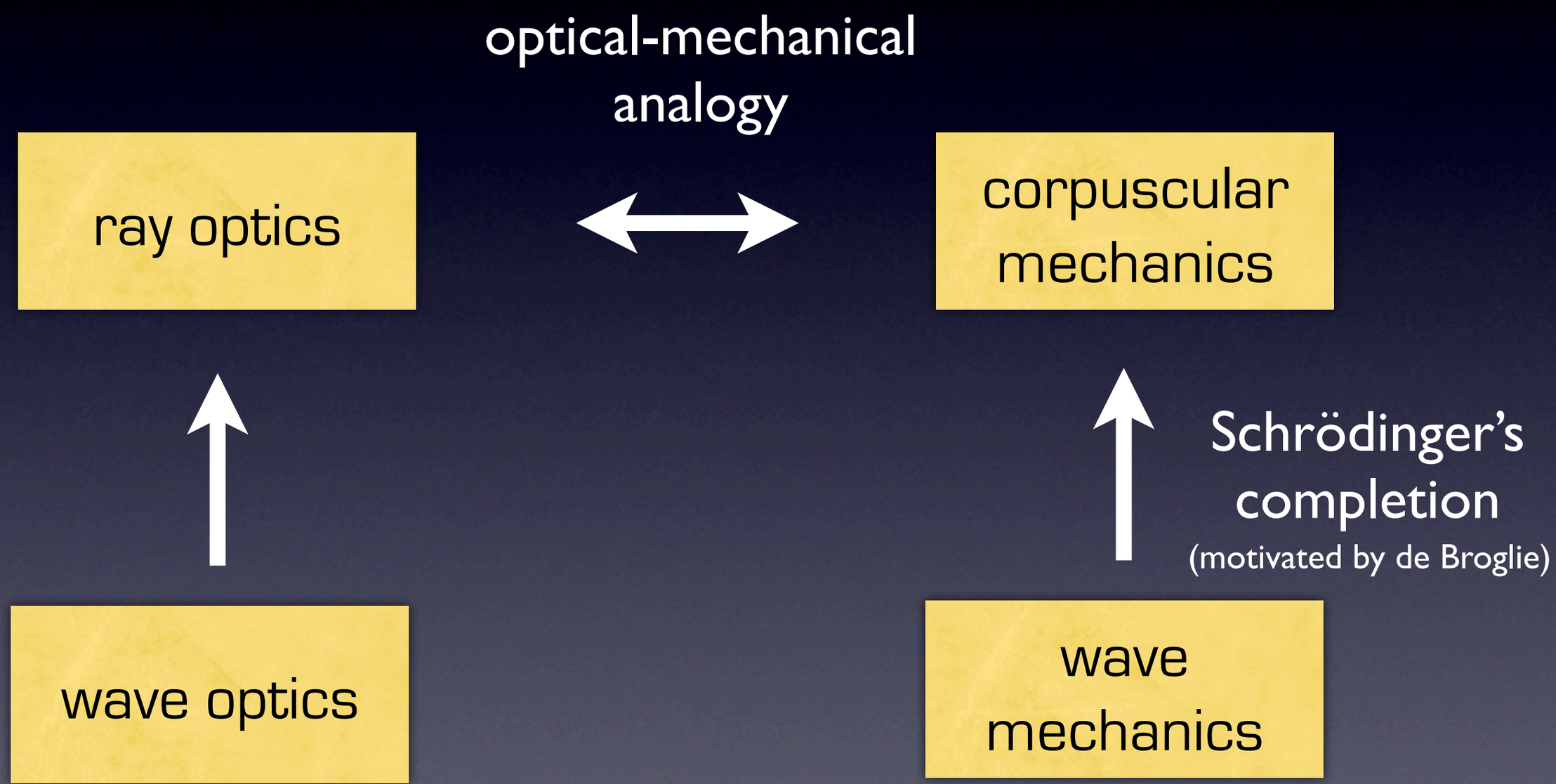
ray optics

corpuscular mechanics

wave optics

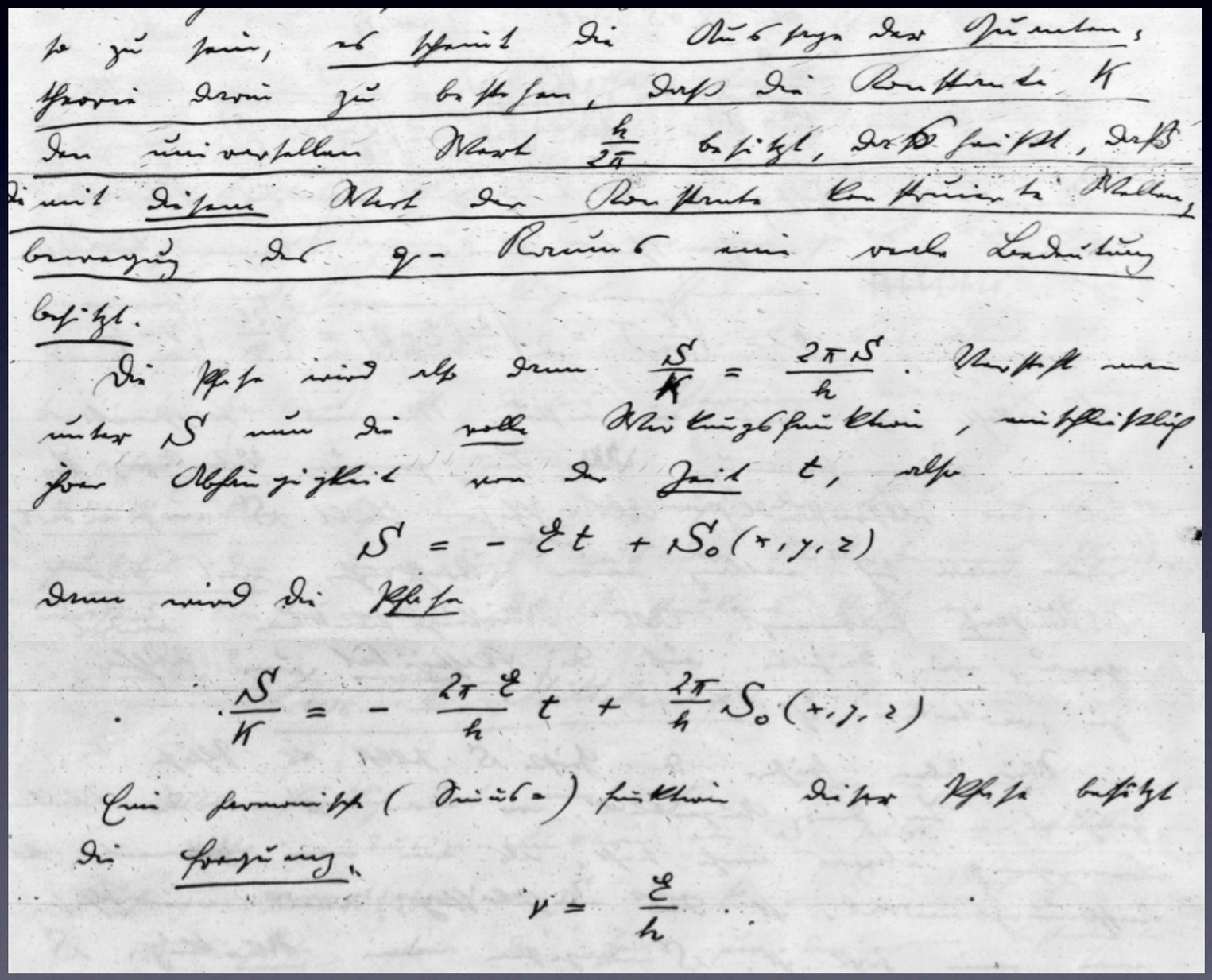
?

Schrödinger's Completion of Hamilton's Analogy



Old quantum theory is the limiting case of a more general wave mechanics!

Schrödinger's Counterpart to the Sharpened Correspondence Principle



[...] the central claim of quantum theory appears to consist in the fact that the constant K has the universal value

$$\frac{h}{2\pi}$$

i.e., that the wave motion in the q-space constructed with this value of the constant has a real significance.

Schrödinger, AHQP 40-7-001 (early 1926)

Conclusion: Pre-established Harmony? The Genetic View:

- Both theories are transformations of a **common ancestor**: old quantum theory!
- Both theories **preserve** the formal structure of Hamiltonian mechanics, while extending just to the right degree.
- Both theories involve a **translation procedure** connecting classical with quantum concepts.
- Both theories incorporate the **new knowledge** about the energy-frequency condition.