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Proximity of Theory and Experimental Practice in Göttingen's Institutes of Physics 1920-1933: Work of Maria Göppert as Exemplar of an Institutional Culture

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The process of science is mediated and influenced by people from various cultural backgrounds, political and economic systems, as well as disparate practices involving different experimental and theoretical approaches. In the 1920's the University of Göttingen was the nexus of theoretical and experimental physics as well as mathematics (Rupke 2002). In this case study of Maria Göppert, a doctoral student under the tutelage of Max Born, we see the effects of the proximity of theory and experimental groups, as well as the roles of people and publications during the period of her dissertation research.

I will explore the impact of proximity and the role of collaboration between theory and experimental practice in the development of quantum mechanics in Göttingen in the 1920's (Kamp et al. 1983). Max Born and James Franck were close friends and colleagues who closely interacted as seen in the dissertation work of Born's graduate students Maria Göppert and Victor Weisskopf (Göppert-Mayer 1931; Weisskopf 1931). Furthermore, I will reconstruct the interactions between theory groups and experimental groups in the advancement of quantum mechanics in Göttingen in the 1920's until 1933. Maria Göppert worked in Born's institute for about a decade; during that period Born's institute hosted a number of important physicists for various periods of time. I will focus on Paul Dirac's 1927 visit to Göttingen and his impact on Maria Göppert's dissertation work. I have reconstructed her dissertation work (*Göttinger Dissertation*) in order to extract the portions that derived from the work Dirac. Additionally, I will show the putative roles of textbooks, monographs, publications, and handbook chapters on the development, codification, and promotion of the quantum mechanics.

1. Physics in Germany at the beginning of the twentieth century

At the beginning of the twentieth century most of the physics departments in German and Prussian Universities were dedicated to experimental work; however, there were courses on theoretical physics. For example, the 1917/1918 volume XVIII of *Physikalische Zeitschrift* (p-500) listed the university calendar for the winter semester with the lectures courses at the University of Göttingen. Peter Debye had lectures on new research in quantum theory, optics for physicists and mathematicians, and Debye and David Hilbert initiated the joint seminar "On the Structure of Matter" (Schirrmacher 2003). Debye collaborated with Hilbert and together with the arrival of Born in Göttingen a plan was initiated to have an institute dedicated to theoretical physics research adjacent to the institute dedicated to experimental physics.

Prior to this initiative there were other physics departments that contained both theoretical and experimental institutes. In Munich, Arnold Sommerfeld chaired the theoretical institute and Wilhelm Röntgen chaired the experimental institute (Heilbron 1967). Beginning in 1906 Sommerfeld led a group of outstanding theoretical physicists that included Max von Laue, Paul P. Ewald, Paul S. Epstein, and Wilhelm Lenz (Seth 2010). It is of note that Sommerfeld also had an experimentalist directly in his group; Walther Friedrich who graduated under Röntgen (Heilbron 1967). Further historical insights and comparisons of scientific culture and style can be obtained from a study of the physics institutes in three cities and their directors: Arnold Sommerfeld in Munich, Max Born in Göttingen, and Niels Bohr in Copenhagen (Eckert 2001).

At first Born was the head of a department in Göttingen. Shortly after in 1921 three institutes were created: Robert Pohl directed the First Institute of Physics, James Franck directed the Second Institute of Physics, and Max Born directed the Institute of Theoretical Physics. Pohl was one of the founders of experimental solid state physics. What is significant is that these institutes were in the main physics building on Bunsenstrasse. The Mathematics Institute was next door to the Physics Institutes. Ludwig Prandtl headed the Institute for aerodynamics research that was on the opposite side of Bunsenstrasse (Hentschel 1999).

Born was not the first professor of theoretical physics in Göttingen. He was preceded by Debye, who in 1914 accepted the newly created professorship in theoretical physics at the University of Göttingen (Schirrmacher 2003). Thus began the series of courses on quantum theory in the physics department. In 1920, when Debye left Göttingen his replacement was the theoretician Max Born. Born arranged for his friend James Franck the experimentalist to become a professor in the adjacent institute. Franck's research was centered on experimental atomic physics.

The scientific collaboration between Born and Franck extended beyond friendship; it was bilaterally synergistic. The productive synergism between the groups of Born and Franck is further described in a recent biography of Franck and in this paper (Lemmerich 2007). As expressed by Gyeong-Soon Im (Im 1995, 74):

> After Born moved from Frankfurt to Göttingen in 1921, he conducted a research program in quantum theory with a distinct style: he selected as simple physical

problems as possible for which there already existed extensive empirical evidence. He then sought general solutions to these problems with the help of rigorous mathematical techniques. Since Franck systematically performed experiments associated with the quantum theory, he accumulated *inter alia* many observational results on quantum excitation during collision processes, including ionization energies of atoms and molecules. Born's close collaboration with Franck was well suited to his research style: a formal and mathematical description of nature based upon plentiful observational data.

2. Maria Göppert as university student and doctoral student

What influences impact on the development of a scientist? Is it family, friends, neighbors, teachers and mentors? Is it primary education and university education? In the scholarly life of Maria Göppert we can trace multiple examples of these influences (Johnson 1999, 2004; Masters 2000, 2008a; McGrayne 1993).

Maria Göppert was born in 1906, she being the only child of Friedrich Göppert and his wife, Maria. The family moved to Göttingen in 1910 where Friedrich Göppert obtained a position as professor of pediatrics. From the late nineteenth century to the early decades of the twentieth century Germany was the center of research in mathematics, chemistry, and physics. One metric is revealing of this prominence in science: until 1933 about 60 percent of the Nobel Prizes in physics and chemistry were awarded to German scientists (Hargittai 2010).

During Maria Göppert's formative years the University of Göttingen was a major center of research in both physics and mathematics. The mathematics faculty included such luminaries as David Hilbert, Richard Courant, Hermann Weyl, Gustav Herglotz and Edmund Landau. During the years of her university studies and doctoral work Maria Göppert came into contact with the following eminent scientists: Arthur Compton, Max Delbrück, Paul Dirac, Enrico Fermi, James Franck, Werner Heisenberg, J. Robert Oppenheimer, Wolfgang Pauli, Linus Pauling, Leo Szilard, Edward Teller, John von Neumann, and Victor Weisskopf (Sachs 1982). Maria Göppert's friends included Max Born, Max Delbrück, James Franck, Linus Pauling, Hertha Sponer, Leo Szilard, and Victor Weisskopf (McGrayne 1993).

Maria Göppert was proximate to this center of intellectual activity and her family was physically and socially connected to many of Göttingen's great intellectuals. For example, the Göpperts lived next door to David Hilbert and they were personal friends. In 1921 Göttingen brought two new physicists to the University; first Born arrived and then Franck. The Göpperts became and remained their good friends. Other family friends included Richard Courant, Edmund Landau, and Hermann Weyl who were in the department of mathematics.

I now explore some of the plausible reasons that may explain why the young Maria Göppert chose to study and then to perform her doctoral research in Göttingen, and then after she earned her doctorate why she chose to fulfill her professional life as a physicist outside of Germany. Years later in 1963, Eugene Wigner, Maria Göppert-Mayer and J. Hans D. Jensen shared the Nobel Prize in Physics (Göppert-Mayer 1948; Göppert-Mayer and Jensen 1955).

First, Maria Göppert fostered a strong interest in mathematics. Göttingen was well known for its outstanding mathematicians and was home to Germany's leading mathematics department (McGrayne 1993; Rowe 1989). Such famous names as George Friedrich Bernhard Riemann, Carl Friedrich Gauss, David Hilbert, Felix Klein, Hermann Minkowski, Richard Courant, Carl Runge, and Emmy Noether gave the University of Göttingen a world-wide reputation. Another attraction for Maria Göppert was that David Hilbert, Richard Courant, and Carl Runge were also interested in physics and mathematical physics. Years earlier when Born studied for both his doctorate and his habilitation in Göttingen, he was influenced by the famous scientists and mathematicians that included Felix Klein, David Hilbert, Hermann Minkowski, Carl Runge, Karl Schwarzschild (full professor of astronomy and director of the observatory), and Woldemar Voigt.

Second, Göttingen and its University of Göttingen had a long and famous standing in liberalism and freedom from censorship. The following is quoted from the Georg-August Universität Göttingen website: (Georg-August Universität Göttingen website, 2010).

> The University bears the name of its founder King George II of Great Britain who, as Georg August, was also Elector of Hanover. In affinity with the spirit of the Enlightenment, Göttingen abandoned the supremacy of theology and set its faculties on an equal footing. A specialty of its constitution is the freedom from censorship as held in writing for the first time in the foundation privilege dated December 7, 1736, according to which the academic teachers at the Georgia

> Augusta …are to have complete and unlimited freedom, access and right to teach publicly and especially for all eternity.

Consistent with this liberal spirit is the story of the "Göttingen Seven" (Lampe 2002; Marchand 1996). In 1837 Dahlmann and the other six protesters demonstrated against any alteration of the constitution of the Kingdom of Hanover. In addition they refused to take an oath to Ernest Augustus, the King of Hanover. They were all dismissed from the university.

Third, the University of Göttingen was home to some outstanding woman and that set a precedent and provided a model for Maria Göppert to pursue her graduate work at that institution. Emmy Noether came to Göttingen in 1916 at the invitation of both Hilbert and Klein and she stayed there until 1934. It was through the efforts of Hilbert, a strong proponent of woman's educational rights that helped Noether to undergo her habilitation and thus gave her the right to lecture at a university. Finally, in 1922 Noether with a doctorate earned thirteen years before was made a Privatdozent; now she could legally teach in the university under her own name. In the course of her second Habilitation lecture she presented her work on invariant forms in mathematics or what is now known as "Noether's Theorem." In 1922 she had the following title unofficial, extraordinary professor, basically a volunteer professor that had no salary nor did she have any pension; in Göttingen she was never made a full professor (McGrayne, 1993, 175-200).

Hertha Sponer who was a friend of Maria Göppert worked on molecular spectroscopy and photochemistry in the laboratory of Franck during the time that Maria

Göppert was a graduate student with Born (Lemmerich, 2007). Sponer studied at the University of Tübingen where she worked as an assistant to James Franck. After one year in Tübingen she moved to the University of Göttingen where she was a doctoral student with her supervisor Debye. She graduated in 1921 with a doctorate; this was a very significant achievement, since she was part of a small group of woman who obtained a doctorate in physics at a German university together with the right to teach at a German University.

Maria Göppert's first plan was to study mathematics which was her strongest interest. Therefore, in 1924 Maria Göppert began her studies in mathematics at the University of Göttingen. Shortly after beginning her studies Born asked her to join the physics seminar and her interest in the newly evolving area of quantum mechanics coupled with her training and interest in mathematics influenced her move from mathematics to physics (Greenspan 2005). When she became a graduate student under Born she already was adept in mathematics and that helped her with the new quantum mechanics. Nevertheless, the strong experimental approach of Franck remained with her during her doctoral research as well as in he later works in the field of nuclear physics (Masters, 2000, 38-41). Maria Göppert was both a close personal friend of Franck and he was a friend of the Göppert family as well. This close relationship would keep her informed of his research projects and their experimental techniques and results.

Maria Göppert worked in Born's institute from 1924 until she graduated in 1931. She continued to return to Göttingen from Baltimore in the summers until 1933 in order to work with Born. Together they published a major review on the dynamic theory of crystal lattices, which appeared in the 1933 edition of the prestigious *Handbuch der Physik* (Born and Göppert-Mayer 1933, 623-794).

Maria Göppert wanted a career in science as a full professor. She recognized that such an aspiration had a very low probability to be fulfilled in Germany. This followed from her knowledge that neither Emmy Noether, nor Lise Meitner, nor did her good friend Hertha Sponer ever achieve a full university professorship in Germany (McGrayne 1993).

Following the April 7, 1933 enactment of the *Law for the Restoration of the Professional Civil Service* almost all non-Aryan civil servants (including tenured university professors) were removed from their positions in Nazi Germany. Born left Göttingen to take a position in the United Kingdom. Franck, the recipient of the 1925 Nobel Prize in Physics, quit his university professorship in a protest against the Nazi racial policies and immigrated to the United States (Lemmerich 2007). In addition to the above two losses, the following were dismissed from their positions in the institutes in Göttingen: Richard Courant, Edmund Landau, directors of the Mathematical Institute; Noether, an algebra instructor; Heinrich Kuhn and Eugene Rabinowitch who were assistants to Franck in the Second Institute of Physics; Walther Heitler and Lothar Nordheim who were assistants of Born, and Edward Teller who was working with Born.

3. Textbooks, monographs, publications, and handbook chapters available to Maria Göppert

I now elucidate the important textbooks, monographs, and handbook articles that influenced the development, codification, and promotion of the quantum mechanics in Göttingen in the 1920's. Sommerfeld's *Atombau und Spektrallinien* [Atomic Structure and Spectral Lines] educated a whole generation of theoretical physicists. The first German edition was published in 1919 and the fifteenth edition was published in 1931.

Now I focus on the impact of the following books from the Göttingen's mathematicians and physicists. Courant and Hilbert's *Methods of Mathematical Physics* was of seminal importance. It was first published in 1924 and a second revised edition followed in 1930. It contained clear expositions of the algebra of linear transformations, the calculus of variations, eigenvalue problems including a section on eigenvalue problems of the Schrödinger type. It served as a mathematical reference for the matrix mechanics of Born, Heisenberg, and Jordan.

Born's monograph *Vorlesungen über Atommechanik* [*Lectures on Mechanics of the Atom*], written with the help of his assistant Friedrich Hund who worked on the quantum interpretation of band spectra of diatomic molecules, contains the lectures that Born delivered in Göttingen during the session 1923-1924. Born credits Hund with writing considerable portions of the text and with providing the mathematical proofs, Heisenberg contributed some sections of the book, and Lothar Nordheim contributed the sections on perturbation theory. Nordheim's sections describe perturbation theory with applications to non-degenerate systems, the non-harmonic oscillator, intrinsically degenerate systems, and the Helium atom in the ground state and in the excited state. The method of separation of variables is not able to solve these problems and Born suggested that instead the successive approximation methods from celestial mechanics could be used to solve problems in the old quantum mechanics.

In 1926, Franck and Jordan completed their textbook, *Anregung Von Quantensprüngen durch Stösse* [*Excitation of Quantum Jumps by Collisions*] (Franck and Jordan 1926). It provided a summary of all the experimental work carried out in Franck's research group up to 1926.

In 1927, Friedrich Hund, a Privatdozent at the University of Göttingen, published *Linienspektren und Periodisches System der Elemente,* which was a seminal book on line spectra and the periodic system of elements (Hund 1927). Line spectra had a seminal role in the development of atomic physics and in the development of the old and the new quantum mechanics. Hund's book provided a consistent approach to the electronic structure of the elements and their line spectra.

Later in 1930, Born in Göttingen and Jordan who was now in Rostock, completed their book *Elementare Quantenmechanik* [*Elementary Quantum Mechanics*] (Born and Jordan 1930). The book is divided into chapters as follows: physical basis, mathematical foundations, the laws of matrix mechanics, angular momentum, perturbation theory, statistical meaning of quantum mechanics, and the introduction to the quantum theory of light. Three appendices described the emission and absorption of electromagnetic waves by classical theory, the Jacobi-Poisson Brackets in classical mechanics, and adiabatic processes. Maria Göppert was well aware of the experiments as well as the content of all of these books as is evident from her thesis publication.

Further evidence for the close interaction between the theoretical and the experimental groups in Göttingen is the fact that Born and Franck co-edited a series of book under the general title of *Struktur der Materie in Einzeldarstellungen* [Structure of matter in monographs]. The first volume of the series was Born's *Vorlesungen Über Atommechanik* that was published in 1925. Other authors in the series and their books included: Peter Pringsheim who wrote *Fluorescenz und Phosphorescenz im Lichte der neueren Atomtheorie*, and Walter Grotrian who wrote *Graphische Darstellung der Spektren von Atomen und Ion mit ein, zwei und drei Valenzelektronen*. These titles and others that were selected for the series span both theoretical and experimental physics and they are indicative of the close professional links between Born and Franck. It is reasonable to assume that Maria Göppert was familiar with these books.

4. What are the roles of other students, visitors, graduate students, and assistants in the development of Maria Göppert's dissertation research?

An important scientific practice is the exchange of scientists between institutions. For example, quantum mechanics as developed in Göttingen in the 1920's was transmitted to educational institutions in the United States predominately by Born's 1925-1926 visit to several American universities and research institutions. How does the exchange of scientists impact on the research practices of graduate students? First, I present the people who were part of Born's and Franck's research groups. Second, I present the impact of Dirac's 1927 visit to Göttingen and his seminal publications that were used in Maria Göppert's dissertation research and publications.

Born’s doctoral students included J. Robert Oppenheimer, Lothar Wolfgang Nordheim, Max Delbrück, Walter Elsasser, Friedrich Hund, Pascual Jordan, Viktor Weisskopf, and Maria Goeppert-Mayer. Born’s first assistant was Wolfgang Pauli, his second assistant was Werner Heisenberg; his other assistants included Léon Rosenfeld and Walter Heitler. Other members of Born’s research team included the following: Erich Hückel, Friedrich Hund, Fritz London, and Eugene Wigner (Born and Born 1962, 12-94; Eckert 2001).

Maria Göppert worked in Born’s institute for about a decade; during that period Born’s institute hosted a number of important physicists for various periods of time: Arthur H. Compton, Enrico Fermi, Linus Pauling, J. Robert Oppenheimer, George Uhlenbeck, Edward Teller, Wolfgang Pauli, Eugene Wigner, Leo Szilard, and Paul Dirac.

Similarly, Maria Göppert would have had the opportunity to interact and exchange ideas with Franck’s students which included Patrick M. S. Blackett, Walter Elsasser, Wilhelm Hanle, Gerhard Herzberg (who also worked with Max Born), Arthur von Hippel, Eugen Rabinowitch, and Hertha Sponer. Visiting scientists that worked with Franck included: Karl T. Compton, Edward U. Condon, Joseph E. Mayer, Gerhard Scheibe, Hans Kopfermann, Werner Kroebel, Heinz Maier-Leibnitz, J. Robert Oppenheimer, and Otto Oldenberg (Lemmerich, 2007).

The 1927 publications of Dirac had a great influence on the Maria Göppert Mayer’s dissertation work (Dirac 1927a, 1927b, 1927c). She directly acknowledged Dirac’s contribution to her dissertation research in her 1931 *Göttinger Dissertation* and in her chapter on dispersion theory in *Elementare Quantenmechanik* (Born and Jordan 1930, 404-408; Dirac 1927b, 1927c). In the section heading of her contribution in *Elementare Quantenmechanik* she cites a footnote that clearly states that the considerations in her section follow from Dirac's paper on the quantum theory of the emission and absorption of radiation and from Dirac's paper on the quantum theory of dispersion (Dirac 1927b, 1927c; Göppert 1930). Furthermore, Born and Jordan in their preface to *Elementare Quantenmechanik* state that Maria Göppert contributed sections on Dirac's theory of emission, absorption, and dispersion (Born and Jordan, 1930, VII-VIII). Besides her own acknowledgment of Dirac's contributions to her research my reconstruction of her *Göttinger Dissertation* provides further evidence in support of my primary claim.

Dirac visited Bohr in Copenhagen from September 1926 through February 1927 where he wrote his paper on transformation theory as well as his paper on the emission and the absorption of radiation by matter (Dirac 1927a, 1927b). Following that visit to Bohr, Dirac remained in Göttingen from February through the end of June 1927 where he wrote his paper on dispersion theory (Bacciagaluppi, Valentini 2009, 84; Dirac 1927c; Kragh 1990, 43).

The following are Dirac's key concepts and assumptions made by Dirac in his two 1927 papers (Dirac 1927b, 1927c). Perusal of Maria Göppert's doctoral research will show the significant use of Dirac's concepts and theory from his papers of 1927 (Dirac 1927b, 1927c; Masters 2010).

Below are the key points of Dirac's paper on the emission and absorption of radiation (Dirac 1927b). He states that the mathematical development in this paper follow from his previous paper on the general transformation theory of quantum matrices (Dirac

1927a). In this paper Dirac procedes as follows (Dirac 1927b). He considers an atom interacting with a field of radiation which is confined to a cavity in order to have a discrete set of degrees of freedom. Dirac considers a finite cavity to enclose the radiation in order to establish a relation between the number of light-quanta per stationary state, and the intensity of the radiation. He restricts the treatment to the non-relativistic case. In the absence of interaction between the atom and the radiation the Hamiltonian consists of two terms: the field and the atom. In the presence of interaction, a third term from the classical theory would be added to the Hamiltonian. From this formulation Dirac derives the correct results for the action of the radiation and the atom on each other. Thus he derives the correct laws for the emission and the absorption of radiation, and the correct values for the Einstein's A and B coefficients (Einstein 1916).

Next I present the techniques and assumptions from Dirac's paper on the quantum theory of dispersion (Dirac 1927c). First I cite some general techniques from Dirac's dispersion paper, in particular those techiques that are relevent to Maria Göppert's research, and then I will give specific examples from Maria Göpppert-Mayer's *Göttinger Dissertation*. In order to help the reader facilitate this posited correspondence I give the page numbers from Maria Göpppert-Mayer's *Göttinger Dissertation* in square brackets which are also included in my English translation of her 1929 paper and her 1931 *Göttinger Dissertation* (Göpppert 1929; Göpppert-Mayer 1931; Masters 2010).

First, Dirac explains that while the new quantum mechanics is used through analogies with classical theory it cannot be applied to a class of problems where the analogies are obscure; for example, the problems of resonance radiation and the widths of spectral lines. Dirac proposes that the radiation field can be treated as a dynamical system composed of harmonic components with energies and phases, where each one is a harmonic oscillator. The interaction of this field with an atom can be described by a Hamiltonian function. Dirac then requires the use of perturbation methods to solve the Schrödinger equation. Dirac then shows through the use of second-order perturbation theory that a double process can occur, first a transition from the initial state to an intermediate state, and then a transition from the intermediate state to the final state; each of these processes does not conserve energy, but energy is conserved in the total process consisting of the two transitions, i.e. from the intial to the final state in a double process. Dirac resolves the electromagnetic field into its components of plane-polarized, propagating waves, with each component of a definite frequency, direction, state of polarization. He confines the radiation to a cavity to discretize the number of components. Then he formulates the Hamiltonian function in terms of a vector potential that describes the interaction of the field with the atom which he considers a single electron in an electrostatic field with a potential. For the case of resonance Dirac assumes a range of frequencies of the incident radiation, and he calculates the equations for the probability of the emission and the absorption of light quanta.

I now elucidate some of the details of his paper on dispersion theory (Dirac 1927c).

1. The basic idea of Dirac's theory of radiation is to describe the total system of the radiation and the atom as the sum of three terms: one represents the energy of the atom, one represents the electromagnetic energy of the radiation field, and the third term represents the interaction energy of the atom and the radiation field. In the absence of the

third interaction, the atom could not absorb nor emit radiation. First, Dirac decided not to consider the radiation in infinite space, but to represent the radiation as confined to a cavity, of finite volume, V, and with perfectly reflecting walls. Later, the cavity will be expanded to become infinite, and that will represent the radiation in free space.Then the oscillations of the confined electromagnetic field are represented as the superposition of a finite number of fundamental vibrations, each one corresponds to a system of standing waves. The electromagnetic field of a monochromatic, plane standing wave in the cavity can be described was a vector potential. Next the Hamiltonian of the atom and the radiation field are described. The electromagnetic energy of the radiation field can be shown to have the same Hamiltonian as a system of uncoupled harmonic oscillators. The Hamiltonian for the total system of atom and radiation field is the sum of three terms: the term for the radiation field, the term for the atom, and the term of the interaction of the radiation and the atom. The last interaction term is the coupling term for the atom and the radiation field. Then Dirac develops his time-dependent perturbation theory to calculate the probabilities of transitions of energy for the atom and for the radiation field. This is studied for a variety of cases: absorption, emission, and induced emission.

2. Dirac uses a semiclassical treatment; the electromagnetic field is treated classically and the atoms with which the field interacts are treated quantum mechanically. The semiclassical approach correctly describes absorption and induced emission, but it fails to correctly describe the influence of the atoms on the electromagnetic field.

3. In the mathematical description of a plane, linear polarized, monochromatic wave that

is resolved into its Fourier components, there appears the frequency of the wave, an amplitude which is a complex vector, and two complex components of the wave amplitude; they are each multiplied by a unit polarization vector, which represent the two independent states of linear polarization.

4. In order to make the number of degrees of freedom discrete and not to become infinite Dirac assumed that the radiation field is confined to a cavity. Dirac's theory the radiation in a cavity can be described by giving the amplitude of each standing wave at a particular time; therefore, the amplitude can be considered as a coordinate that follows the laws of quantum mechanics. In his theory the interaction of atoms and radiation he calculated the probabilities of both induced emission and spontaneous emission [no radiation present]. In addition, it provided a new theory for dispersion and light scattering.

5. In the treatment of an atom and its interaction with a radiation field, the process of the absorption of a photon by an atom involves the increase in the energy of the atom by a quantum of energy, and the decrease of the harmonic oscillators comprising the radiation field by a quantum of energy. The combined energies of the electron and the radiation oscillators follow the law of conservation of energy.

6. Dirac's perturbation theory included two cases: time-dependent and time-independent perturbations. An example of the former case is the calculation of absorption of light or the induced emission of light by an atom in a radiation field.

7. Dirac's time-dependent perturbation theory can be used to calculate transitions between discrete energy levels as well as in physical systems with continuous energy levels. For example, in particle collisions, the eigenfunctions of the free particles, i.e. colliding electrons with atoms, are described as plane waves, and the energy of the particles is not quantized, but can take different positive values. If the particles are now confined to a box the eigenvalues or the energy of the particle is not quantized. As the side of the box increases to infinity, the free particle eigenfunctions and energy eigenvalues approach those of the free particle. For a free particle in a box, the quantized energy eigenvalues can be calculated by perturbation theory for discrete energy levels. Then the size of the box is increased to infinity, and the result obtained is valid for continuous energy levels.

8. Raman scattering is another example of a two-photon process; a photon is absorbed and another photon is emitted and the atom makes a transition from the initial to the final state. The energy difference between the initial and the final states is equal to the energy difference of the two photons. Second-order perturbation can be used to calculate the Raman transition probabilities. Transition probabilities are the square of the transition amplitudes for the process. Time-dependent perturbation is required to calculate the rates of transitions.

9. Dirac states that the exact interaction energy of the field and the atom is too complicated, therefore he uses the dipole energy. That approximation results in a divergent series that appears in the calculation. In his calculation of dispersion and

resonance radiation there is no divergent series, but when he attempts to calculate the breadth of a spectral line a divergent series appears.

As we shall see in the following section many of these aspects of Dirac's 1927 publications were directly incorporated into Maria Göppert's *Göttinger Dissertation*. Next I refer to Maria Göppert-Mayer's 1931 *Göttinger Dissertation* and point out the many congruences of her work with the previous research as described in Dirac's dispersion paper (Dirac 1927c; Masters 2010).

5. Reconstruction of Maria Göppert's *Göttinger Dissertation*

The origin of Maria Göppert's dissertation research were the two publications of Oldenberg and Franck on electronic excitation of atoms due to inelastic collisions with electrons and the subsequent luminescence (Oldenberg 1928; Franck 1928). The significance of these important experiments is that they demonstrated the discrete energy levels of atoms. The inelastic collisions of electrons and atoms can result in the transfer of energy to the atoms and can excite the atoms without ionizing them. These experiments were conducted at the Second Institute of Physics and they provided Maria Göppert with an opportunity to seek a theoretical basis to explain these findings.

Next I proceed to briefly review the paper of Oldenberg and the paper of Franck (Franck 1928; Oldenberg 1928). The basis of his experiments was the question: could an atom become excited [its electrons in higher energy states than the ground electronic state] through the working together, in a single act, of collisions with electrons and an incident light field? He also discussed the concept that two light quantum can work together in one elementary act to excite an atom or molecule, i.e. the Smekal-Raman effect (Smekal 1923).

Oldenberg produced experimental evidence on the broadening of the resonance lines of mercury atoms when the excited atoms collided many times with slow particles. He showed that the excitation energy of the mercury atoms can be transferred as kinetic energy to the particles, and the difference frequency is radiated as light. The publication contains an equation that show how two light quanta, with two different frequencies can work together in a single elementary act to excite an atom [double absorption or two-photon absorption]. In the second section of Maria Göppert's *Göttinger Dissertation* she worked out the theory of "the working together of light and collisions [electrons] in one elementary act." Her theoretical analysis agrees with the previous experimental results of Oldenberg (Göppert-Mayer 1931, 288; Oldenberg 1928).

Franck focused his research program on atomic physics and spectroscopy. In 1925 Franck was awarded the Nobel Prize in Physics for his earlier work during the 1912-1914 period, specifically the Franck-Hertz experiment that was based on the inelastic scattering of electrons by mercury atoms in the gas phase. Franck and Hertz demonstrated that a collision between an electron and atom can result in a transition of the atom from its ground state to a stationary state of highter energy; in the process the electron loses an equal amount of energy (Franck and Hertz 1914). Their experiment provided important confirmation of the quantization of an atom's energy levels.

In Göttingen, Franck continued to experiment with collisions of fast electrons and atoms. He explored the effect of the velocity of the colliding electrons on the spectral lines of the atoms. He studied the ionization of atoms due to collisions with slow and fast electrons and the subsequent luminescence that was observed. He interpreted this process as due to the recombination of ions and electrons (Franck 1928).

In order to obtain a sense of the physical theories and techniques that were in use at the time of Maria Göppert‘s graduate research I recommend that the reader study the book *Elementare Quantenmechanik* (Born and Jordan 1930). Maria Göppert contributed a section to this book on Dirac's theory of emission, absorption, and dispersion which is similar to her thesis work, and the authors acknowledged her contribution in their foreword. Note that in her 1929 paper she stated that Dirac's dispersion theory described not only the Raman Effect, but the reverse process in which two photons act together in a single elementary act to promote an atom from the ground state to an excited state (Dirac 1927c; Göppert 1929). In her subsequent more complete calculation published in 1931 of two-photon processes, she used the Dirac formulation of the time-dependent perturbation theory (Dirac 1927c; Göppert-Mayer 1931). In addition, it is useful to read her fellow graduate student Victor Weisskopf‘s *Göttinger Dissertation* on resonance fluorescence, (Weisskopf 1931). At the end of his *Göttinger Dissertation* Weisskopf thanked Born, Franck, and Wigner for many supportive suggestions and discussions which provides further evidence of the interactions between the experimental group headed by Franck and the theoretians Born and Wigner (Weisskopf 1931).

Maria Göppert worked on the theory of atom-photon interactions. Building on the dispersion theory of Kramers and Heisenberg, and the time-dependent perturbation theory of Dirac, she developed analytical expressions of the transition probability for multiphoton absorption and stimulated emission as well as Raman scattering processes (Kramers and Heisenberg 1925).

What theoretical and mathematical techniques did Maria Göppert use in her dissertation research that followed the previous publication of Dirac (Dirac 1927c). In particular a careful analysis of her dissertation paper reveals the following similarities with Dirac's dispersion paper. The first section of her dissertation is concerned with two light quanta working together in one elementary act (Göppert-Mayer 1931, 273-284). I cite the page numbers of these similarities in brackets that refer to her *Göttinger Dissertation* that was published in *Annalen der Physik* (Göppert-Mayer 1931). The quotations are from my English translation of her *Göttinger Dissertation* (Masters 2010).

(1) With the help of Dirac's dispersion theory, the probability of an analogous Raman effect process is calculated, namely the simultaneous emission of two light quanta. It is shown; that a probability exists for an excited atom to divide its excitation energy into two light quanta…. If an atom is irradiated with light of a lower frequency than the frequency associated with an eigenfrequency of the atom, additionally there occurs a stimulated double emission…. Kramers and Heisenberg [1925] calculated the probability of this last process in a corresponding manner [p. 273].

(2) The reverse process is also considered, namely the case that two light quanta, whose sum of frequencies is equal to the excitation frequency of the atom, work together to excite the atom. It is further investigated how an atom responds to colliding particles, when at the same time it has the possibility of spontaneously emitting light. Oldenberg (1928) experimentally found a broadening of the resonance lines of mercury, when he allowed the excited atoms to collide many time with slow particles [p. 273]. For this process, an

> equation is derived here that is analogous to the Raman effect or double emission [p. 274]. Finally, in relation to a study by James Franck (1928), an attempt is made to explain the behavior of the intensity of excitation of spectral lines, induced by collision [of atoms] with fast electrons in such a double process [p. 274]. The calculation shows a probability for such a process, the nature of which will be discussed [p. 275].
>
> (3) The following calculation is closely associated with the work of Paul Dirac on emission, absorption, and dispersion [p. 275].
>
> (4) Let us consider the interaction of an atom with a [electromagnetic] radiation field. To make the number of degrees of freedom countable, think of the radiation contained in a cubical box of volume V, which constrains the light waves to the condition of periodic repetition [standing-waves]. Later this box will be assumed to be infinitely large. Such a radiation field is equivalent to a system of uncoupled harmonic oscillators. The radiation can be decomposed in plane, linear polarized waves, let A be the vector potential… [p. 276].

Perusal of her *Göttinger Dissertation* indicates that Maria Göppert made use of the following assumption and techniques:

(1) the confinement of the radiation field in a cavity so that the number of the degrees of freedom can be discrete and not become infinite,

(2) the use of the vector potential [p. 277],

(3) the description of the total Hamiltonian function consisting of three components: the Hamiltonian of the radiation field (the uncoupled harmonic oscillators), the Hamiltonian of the atom, and the Hamiltonian of the interaction between the atom and the radiation field [p. 277], the electric dipole approximation in which it is assumed that the wavelength of the light is much larger that the atom's diameter, this is the assumption that the electromagnetic field is constant over the atom's diameter [pp. 277-278],

(4) the use of second-order, time-dependent perturbation theory [pp. 278-284],

(5) the use of two-photon transitions via virtual intermediate states [p. 278-284], and

(6) Maria Göppert mentioned the "method of variation of constants" [p. 280]. The state of an atom is represented by an expansion in terms of the unperturbed energy eigenfunctions. The Hamiltonian operator is different from the true Hamiltonian by a very small term that is the perturbation. It is called the *method of variation of constants* because the constant coefficients used in the expansion of the wave function in terms of the true energy eigenfunction vary with time.

The second part of Maria Göppert's *Göttinger Dissertation* is the working together of light and collisions [electrons] in one elementary act (Göppert-Mayer 1931, 284-294). In this part she used many of the same techniques and approximations that she used in the first part. First, she defines the Hamiltonian function of the total system in which the interaction energy is separated into two parts; one term is the interaction of the atom and the radiation, and the second term is the interaction between the atom [nucleus] and the electron that is approximated by the Coulomb field. The electron waves are enclosed in a cavity with the same conditions as for the radiation; periodic standing waves. In the first case she assumes only one atom and one electron in the cavity and no radiation, thus there are only emission processes. She calculated the probabilities for transitions in the state of the atom due to light alone, and also a similar calculation for the transitions due to electron collisions alone. Then he uses second-order perturbation theory to study the working together of both light and collisions. The second part of her *Göttinger Dissertation* was stimulated by the experimental results of Franck's research group and it confirmed many of their findings (Göppert-Mayer 1931, 284-294).

In summary, the proximity of the theory and the experimental groups as well as the visitors to Göttingen, in particular the 1927 visit and publications of Dirac, all contributed to the theoretical research of Maria Göppert during her doctoral period. Perusal of Dirac's 1927 papers and her 1929 and 1931 publications indicate the developments of Dirac that were incorporated into her dissertation research and publications (Dirac 1926, 1927a, 1927b, 1927c; Göppert, 1929, 1931; Masters 2010).

6. What was known and what did Maria Göppert contribute in her dissertation research?

My studies of the *Göttinger Dissertation* of both Maria Göppert and Victor Weisskopf raised the question of the level of originality that was required at that time for a doctoral dissertation. Both dissertations are approximately of the same level of originality. It is important to understand the role of the dissertation and the habilitation to put this question of originality into a correct perspective. In Germany and other European countries, before a person with a research doctorate could teach in the university they had to obtain a habilitation which gave them this right. The habilitation research differs from the research doctorate; while the research doctorate is performed under the supervision a guiding professor, the habilitation research is based on independent scholarly work. In general, the level of scholarship for the habilitation is significantly higher as compared to the research doctorate.

The *Göttinger Dissertation* of Maria Göppert relied on second-order, time-dependent perturbation theory. Since perturbation theory was a major mathematical technique in her doctoral theoretical research it is necessary to look into its antecedents. What are the sources of this theory and how did approximation methods from celestial mechanics find a place in quantum mechanics?

The early development of these perturbation techniques derived from problems in astronomy (Masters 2008b 36-41). In order to solve three-body problems or n-body problems the following techniques were developed. For the case when the Hamiltonian for the exact problem is known, and it only slightly differs from the Hamiltonian for the more complex insoluble problems, then approximation or perturbation techniques were derived. The fundamental basis of all the perturbation methods is that the solutions of the perturbed system are only slightly different from the solutions (integrated form of the equations of motion) of the equations of motion of the unperturbed systems that are already integrated. The main mathematical problem to overcome is that when series expansions were used as approximations of a function, they did not always converge or sum to a finite term; in many cases they diverged to infinity (Born 1924, 1925).

In the winter semester of 1922-1923, Born arranged to give a course on perturbation theory at his institute in Göttingen. In 1922 Paul Epstein independently developed his form of perturbation theory with applications to quantum mechanics (Epstein 1922a, 1922b). Earlier in 1916 Epstein developed a perturbation method to treat the helium atom (Epstein, 1916). His method was based on similar work by the French astronomer Charles-Eugène Delaunay. Born recognized that the perturbations in his theory were similar to the degenerate perturbations in celestial mechanics that were called "secular perturbations" (Born 1924, 1925).

Much of the later progress on perturbation theory stems from the works of Born, Schrödinger, Epstein, and Dirac; these methods built on the earlier work of Poincaré. The early formulations of perturbation theory were modified for their application in both the old and the new quantum theories (Masters 2008, 36-41). In 1926 Schrödinger published five papers on his newly derived wave mechanics and some applications to the Stark effect of the Balmer lines. He developed his time-dependent wave equation and was able to calculate the intensities and polarization of the Stark effect on the Balmer series of transition in the hydrogen atom. His expression for the energy shifts is equivalent to that derived by Epstein. In 1926 and 1927 Dirac developed his time-dependent perturbation theory (Dirac 1926, 1927). Dirac's time-dependent theory was the basis for the work of Maria Göppert in her doctoral research.

Prior to Maria Göppert's doctoral research there were previous experimental observations that are examples of two-photon transitions that occur under the combined action of radiation and another perturbation. For example, the experimental studies of Oldenberg and Franck that are cited in the 1931 *Göttinger Dissertation* of Maria Göppert. In the second part of her *Göttinger Dissertation*, she calculated the probabilities of the combined action of light and electron collisions in the transition of atoms.

Maria Göppert's *Göttinger Dissertation* contained the theoretical basis for two-photon absorption and emission processes; she called the effects "double absorption" and "double emission." The probability of the two-photon process is proportional to the square of the light intensity, and the rate constant for the two-photon process is very low compared to a single-photon process that has a rate constant proportional to the light intensity. She predicted nonlinear interactions between light and matter mediated by multiphoton processes. Furthermore, she showed in her *Göttinger Dissertation* that in a double transition or a two-photon transition via intermediate states or virtual state, each part of the transition does not obey the conservation of energy law; however, the total transition from the initial state to the final state follows the law of conservation of energy. Since the double or two-photon transitions are related to the square of the intensity of light they are very improbable with the light sources available prior to the development of high power pulsed lasers (Maiman 1960).

Weisskopf and Maria Göppert were contemporary doctoral students of Born in Göttingen. It is interesting to explore Dirac's influence on Weisskopf's research and to compare Dirac's influence on the two doctoral students. Persual of Weisskopf's *Göttinger Dissertation* and his biography provide additional support for the role of proximity of the experimental and theory groups as well as the effect of the visitors on both Maria Göppert's and Weisskopf's research programs (Weisskopf 1931, 1991).

Although Weisskopf arrived in 1928, each of them published their *Göttinger Dissertation* in the same 1931volume of *Annalen der Physik*. Weisskopf in his biography cites the people who had seminal influences on his research in Göttingen: Franck the experimental physicist who could accurately predict the results of an experiment or a theoretical calculation; Hilbert and especially Courant who taught Weisskopf advanced mathematics; the three young teachers Heitler, Nordheim, and especially Herzberg who taught a course on “Introduction to Quantum Mechanics” that included the latest developments (Weisskopf 1991). In addition to these influential teachers, in 1929 Paul Ehrenfest came to Göttingen from Leiden in order to take over Max Born’s classes following Born’s stroke.

According to Weisskopf's biography it was the 1927 paper of Dirac, “The Quantum Theory of the Emission and the Absorption of Radiation,” which was published prior to Weisskopf's arrival in Göttingen, that influenced Weisskopf's choice of a thesis problem (Weisskopf 1991). Diracs’s paper demonstrated how to calculate the rates of the emission and absorption of light from an atom, but not how to calculate the line width of the transitions; Weisskopf decided to investigate the line width shapes for the transition from the first excited state to the ground state.

Born had a stroke shortly before Weisskopf came to Göttingen in 1928; therefore, Weisskopf turned to Wigner as a mentor. Wigner often visited Gottingen from Berlin. Together, they started with Paul Dirac’s 1927 paper on radiation and developed a novel theory that was published in two papers in 1930 (Weisskopf and Wigner 1930a, 1930b). Their first paper was entitled: “Calculation of the natural line width due to the Dirac theory of light” (Weisskopf and Wigner 1930a). It cited Dirac’s 1927 radiation paper two times (Dirac 1927b). In their paper the authors credit Dirac for previously publishing the techniques used in their present calculations for the interaction of light and matter. These include standing waves of radiation in a cavity and the matrix methods to calculate transitions. The authors also wrote a footnote crediting Maria Göppert for a similar calculation that she published in her 1929 paper (Göppert 1929).

In their second paper Weisskopf and Wigner extended their calculation of the natural line width due to the Dirac theory of light interacting with an atom. The authors found that their quantum mechanically calculated line width of a harmonic oscillator coincides perfectly with the line width as calculated by the classical theory (Weisskopf and Wigner 1930).

Weisskopf and Wigner's two 1930 publications were important since it was the first example of a theoretical problem, the line-width of a harmonic oscillator, that was solved by quantum mechanics without the use of perturbation theory; this so-called Weisskopf-Wigner theory was later used to solve other problems in quantum field-theory (Weisskopf 1991). Because this joint research could not be submitted as his dissertation work, Weisskopf used the same theoretical approach to solve the problem of the re-emission of light absorbed from atoms. The title of his *Göttinger Dissertation* is: *Zur Theorie der Resonanzfluoreszenz* (Weisskopf 1931). Weisskopf's selection of this topic was also influenced by the work of Robert Wood, an experimental physicist who worked at Johns Hopkins University, in Maryland and published spectroscopic data on resonance fluorescence (Wood and Ellett 1924). Weisskopf discussed Wood's spectroscopic studies with Franck whose spectroscopic group was involved with measurements of line-widths. In summary, study of the *Göttinger Dissertation* of Maria Göppert and that of Victor Weisskopf indicate the seminal influences of Dirac's prior publications and also the mutual influences of the theory and the experimental groups in Göttingen's physics institutes.

7. Conclusions

Ancillary evidence of the interactions between the theory and the experimental groups of Born and Franck in the late 1920's is provided by the dissertation research and subsequent biography of Weisskopf, a student of Born and a contemporary student and friend of Maria Göppert (Weisskopf 1931, 1991). From the previous discussion I conclude that although Dirac's 1927 publications were known to both Maria Göppert and to her fellow doctoral student Victor Weisskopf, their *Göttinger Dissertation* of 1931, and their immediate prior publications of 1929 and 1930 respectively, demonstrate that Maria Göppert's research borrowed more heavily (with appropriate citations to Dirac) then Weisskopf did. Both Maria Göppert and Weisskopf were aware of the experimental spectroscopic studies on atoms that were performed in Franck's institute and those studies influenced their choice of dissertation topics to investigate for their dissertation research. There was a substantial impact of the proximity of theory and experimental practice in Göttingen's institutes of physics from1920 to 1933.

Maria Göppert's research on two-photon processes derived from the experimental studies Oldenberg and Franck, the publications and books available to her; in particular, Dirac's six month visit to the Physics Institute and his paper on dispersion. She calculated the transition probabilities for two-photon absorption, two-photon emission, and two-photon Raman processes for the Stokes and the anti-Stokes cases. Subsequently her theoretical dissertation research verified the previous experimental findings of Oldenberg's and Franck's 1928 papers. With the invention of the laser her theoretical predictions were verified and resulted in the expansion of the field of nonlinear optics and its manifestations in modern biomedical optical imaging (Maiman 1960; Masters and So 2008).

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