

**The Resolution of the Particle-Wave Dualism:
Pascual Jordan and the
Quantum Field Theory Program**

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QUANTUM
HISTORY
PROJECT





Pascual Jordan
(1902 - 1980)

"The unsung hero among the creators of quantum mechanics" (S. Schweber)

Early visionary of the quantum field theory program.

- What was this program?
- How did it develop?
- How did it relate to quantum mechanics?

Jordan's views also offer a new perspective on the history of quantum mechanics.

The fluctuation of radiation

Einstein 1909: formula for the fluctuation of the energy of electromagnetic radiation from Planck's radiation law and Boltzmann's law $S = k \ln W$.

$$\langle \Delta E_\nu^2 \rangle = h\nu \langle E_\nu \rangle + \frac{c^3}{8\pi\nu^2\delta\nu} \frac{\langle E_\nu \rangle^2}{V}$$



particle fluctuation term
(from Wien law)



wave fluctuation term
(from Rayleigh-Jeans law)

The formula (somehow) expresses the duality of particle and wave nature of light. But what theory could yield such a dual fluctuation remains a mystery.

Jordan's dissertation

Jordan arrives in Göttingen in 1922—just in time for “Bohr Festspiele.”

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Jordan's dissertation

Jordan arrives in Göttingen in 1922. Works with Born on lattice dynamics, with Courant on "Methoden der mathematischen Physik," with Franck on "Anregungen von Quantensprüngen durch Stöße."

Turns to quantum theory of radiation in his 1924 dissertation "Zur Theorie der Quantenstrahlung."

Thorough study of Einstein's arguments about the statistics of the interaction of light and matter.

Tries to evade Einstein's 1916 conclusion of directed emission of light quanta. Einstein refutes the argument.

Waves and particles

1924/25 Einstein's quantum theory of the ideal gas based on Bose statistics: Close analogy between statistics of light and material gas. New "quantum" statistics for both.

Einstein shows that Bose statistics leads to dual fluctuation term. Refers 1925 to de Broglie's idea of matter waves as a possible explanation.

Schrödinger on waves and particles

Einstein's reference to de Broglie leads Schrödinger to wave mechanics: The new statistics implies that matter should not be seen as consisting of individual independent particles, but as consisting of waves.

“This means nothing else but to follow through with the de Broglie-Einstein undulatory theory of the moving corpuscle, according to which the corpuscle is nothing more than a sort of ‘foam crest’ on a radiating wave which forms the substratum of the world.”

Jordan on waves and particles

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In 1925 (still before Heisenbergs 'Umdeutung'), Jordan draws a different conclusion: Treating the equilibrium of light and B-E gas, he stays much closer to Einstein's own formulation ("a far-reaching formal similarity between radiation and gas"). "The elementary acts of dispersion [between radiation and matter] can be viewed not only as dispersion of light radiation on material corpuscles but also as dispersion of matter radiation on corpuscular light quanta; therefore, the probabilistic law will be symmetric. . . [between the densities of radiation and matter]." (Jordan 1925)

The principle of symmetry of representations.

This means that **both** matter and radiation can be described equivalently **either** as waves **or** as particles. Unlike Schrödinger, no physical model, just a formal symmetry. (Preexistent hostility to 19th century materialism as a possible motivation.)

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Jordan refers to William Duane's 1923 quantum model of interference as inspiration. Interference on a grid can be described as quantized scattering.

Jordan wants to find a theory representing this symmetry. (In a later interview, Jordan claims that he already here thought about a quantum field theory.)

Quantizing the electromagnetic field

After Heisenberg's 'Umdeutung' paper, Born and Jordan develop matrix formulation.

In the 'Dreimännerarbeit', Jordan applies matrix mechanics to the electromagnetic field. He is able to derive Einstein's dual fluctuation formula.

"...almost the most important thing I have contributed to quantum mechanics."

The vibrating string

Actually a simplified model: one-dimensional string. Introduced by Paul Ehrenfest in a 1924 Göttingen talk to discuss the problem of fluctuations black-body radiation. Vibrating string fixed at both ends. Its total energy is constant. But the energy of a small segment of the string fluctuates because of interference of neighboring wave modes “breathing in and out energy.” Calculating this fluctuation can be done by an integration. Relatively simple, since each wave mode is an independent harmonic oscillator. Classical calculation gives only the wave fluctuation term.

The resolution of the wave-particle duality

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Implication: Matrix mechanics extended to the treatment to the electromagnetic field solves Einstein's riddle of the dual fluctuations.

- Problem of infinities.
- Skepticism about field quantization from his colleagues, including Heisenberg and Born.

Quantized matter waves?

However, Jordan goes even further: Sees field quantization as the way to treat the many-particle problem (or at least says so in 1927).

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Not implausible:

- Symmetry of representations implies that matter and light should obey the same fundamental laws.
- From Einstein's gas theory Jordan has learned that it is not enough to make a quantum theory of statistically independent particles.
- Rather, 'quantizing' matter waves is the way to get the correct quantum theory of several particles.

Wave mechanics

Spring 1926: Schrödinger's wave mechanics

Jordan's reaction (letter to Schrödinger, summer 1927): „Then your hydrogen paper gave hope that by following up this correspondence also the non-ideal gas could be represented by quantized waves—that therefore a complete theory of light and matter could be derived in which, as an essential ingredient, this wave field itself operates in a quantum, non-classical way.“

Many many-particle theories

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- Heisenberg and Dirac: phenomenological arguments for symmetrical/antisymmetrical states.
- Schrödinger: self-interaction of continuous charge distribution (Heitler/London).
- Jordan: quantized waves.

Heisenberg/Dirac approach is quickly successful, which changes the landscape: no need for Jordan's much more abstract line of argumentation.

Jordan's program has problems: Fermi-Dirac statistics, general skepticism from colleagues.

Transformation theory

In 1926/27 Jordan and Dirac unify matrix and wave mechanics into transformation theory.

For Jordan, this is the formalization of his principle of symmetry of representations: Particle and wave picture correspond to position and momentum basis. Neither is fundamental.

Rather, the fundamental concept is an abstract probabilistic transformation amplitude.

Sees that as empirical confirmation of positivism in the sense of Mach.

Conspicuous absence of the concept of a physical state (as opposed to von Neumann).

Against visualisation

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Substantial argument: Jordan agrees that light and matter show analogous behavior and should be treated analogously in quantum theory.

However, since the light quantum effects show that classical wave optics fails, therefore wave mechanics can also not be the complete story for matter.

Otherwise, there would be a disanalogy.

Wave mechanics is only a wave theory for the one-particle case. Schrödinger's visualization is misleading.

Dirac's theory of radiation

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- Dirac "quantizes" not the field, but expansion coefficients in a perturbation expansion $\psi = \sum_r a_r \psi_r$
- Shows that in this case $a_r^\dagger a_r$ can be interpreted as number of particles, which obey Bose-Einstein statistics (comparison with symmetricized wave function).
- No direct connection of a_r with electromagnetic field strengths. Only indirect comparison through energy density of field vs. particles.

Dirac's warnings

Rather:

- Dirac explicitly denies that the “wave function of the light quanta” is the same as the electromagnetic field. (Amplitudes have a different meaning.)
- Therefore, the quantization procedure is not an explanation of the quantum nature of radiation. It is only an elegant way to take into account the Bose statistics of light quanta.
- The procedure is not applicable to electrons, since they do not obey Bose statistics. Electrons only have a wave function, but not a physical wave field.

Unlike Jordan's earlier attempt, Dirac's theory is greeted with enthusiasm, since it offers the first autonomous representation of interaction of matter and radiation in quantum mechanics.

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- Jordan treats “second quantization” of the Schrödinger wave function as the quantization of a physical field.
- Jordan sees that as an explanation of the particle character of matter and light: He postulates that particles **are** quanta.
- Jordan shows that it is possible to obtain Fermi statistics from field quantization.

Jordan's vision

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Paper is flawed and rewritten half a year later with Eugene Wigner. Only then formulation of anticommutation rules. But already the first paper defines Jordan's program: a unified quantum field theory for matter and radiation. Particles and waves are only complementary aspects of the underlying quantum field.

Electrons explained

“Despite the validity of the Pauli instead of Bose statistics for electrons, the results achieved so far leave hardly a doubt that [...] the natural formulation of the quantum theory of the electron will have to be achieved by comprehending light and matter on equal footing as interacting waves in three-dimensional space. The fundamental fact of electron theory, the existence of discrete electrical particles, thus manifests itself as a characteristic quantum phenomenon, namely as equivalent to the fact that matter waves only appear in discrete quantized states.”

The progress of the program

In the summer of 1927, Jordan only has a theory of the nonrelativistic free field (for bosons and fermions). He very quickly makes further steps towards a full quantum electrodynamics:

- Jordan and Pauli (summer 1927): Formulation of relativistically invariant commutation rules for the free field. Analysis of quantized field strengths as functional operators.
- Jordan and Klein (fall 1927): Quantum field theoretical reformulation of instantaneous interaction between particles. Introduction of local field operators $\Phi(r)$ as Fourier transforms of amplitudes a_k .

Rather suddenly, Jordan's furious productivity stops. Full treatment of quantum electrodynamics is given by Heisenberg and Pauli in 1929.

Quantum **mechanics** according to Jordan

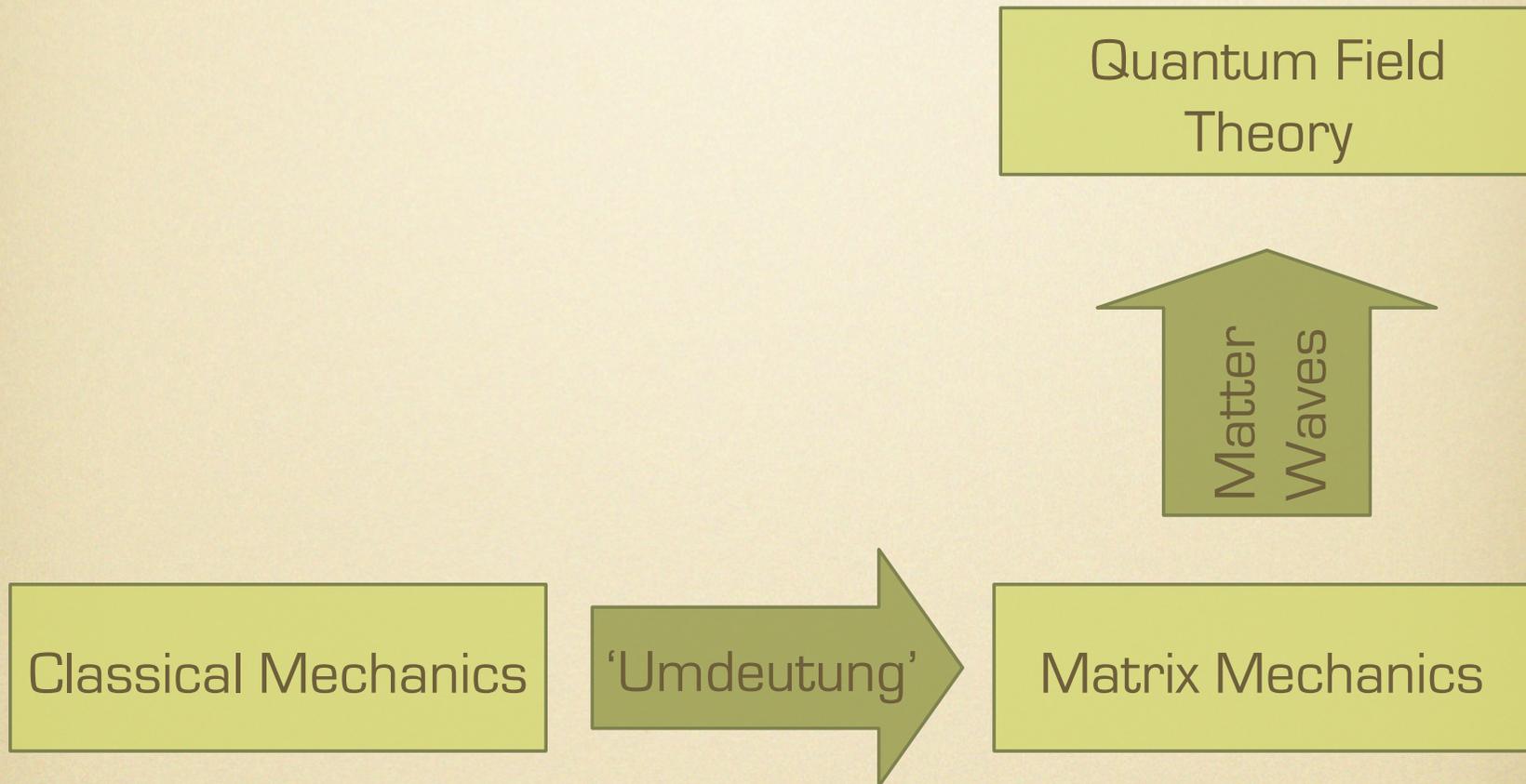
Classical Mechanics

'Umdeutung'

Matrix Mechanics

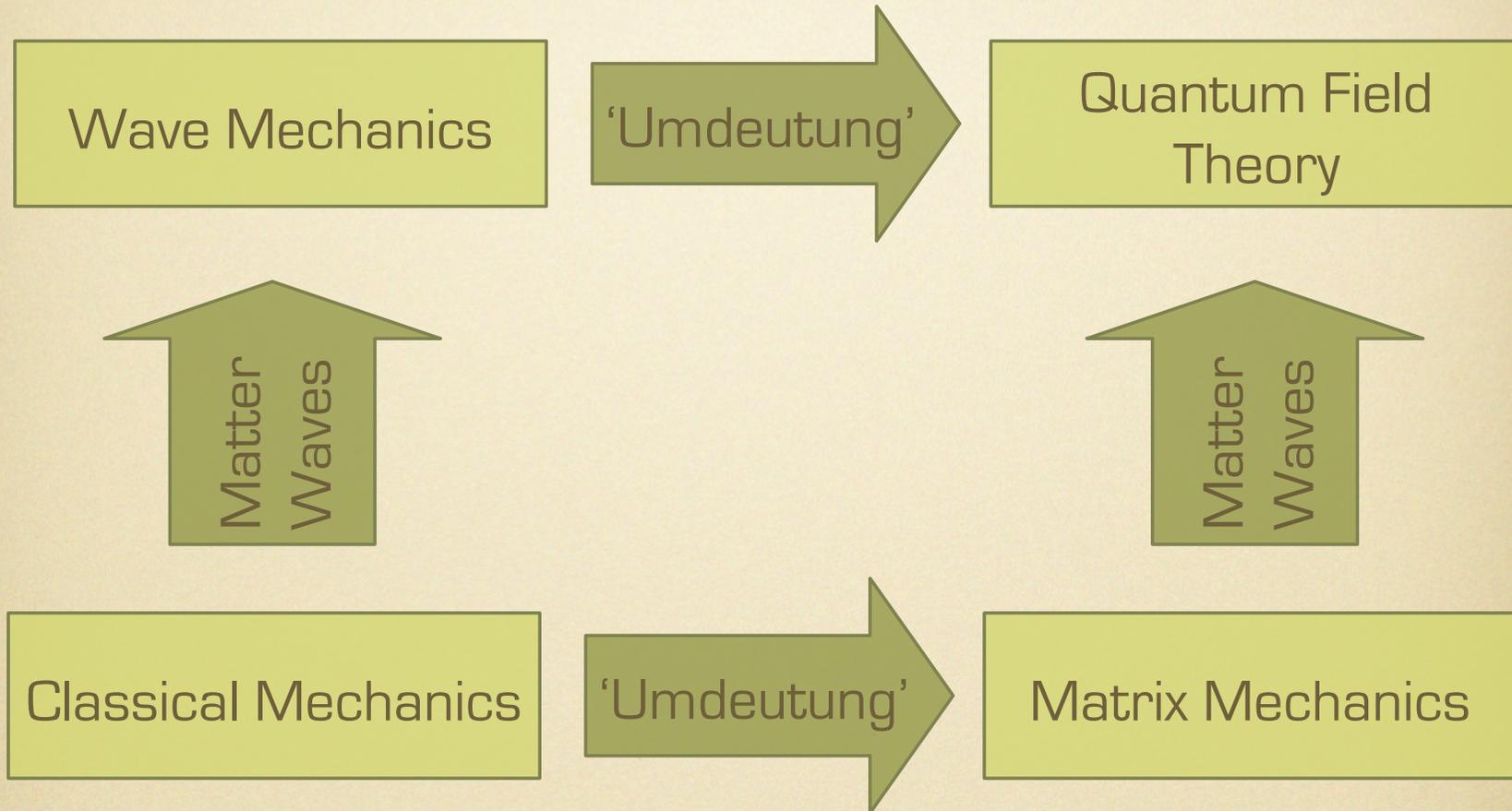


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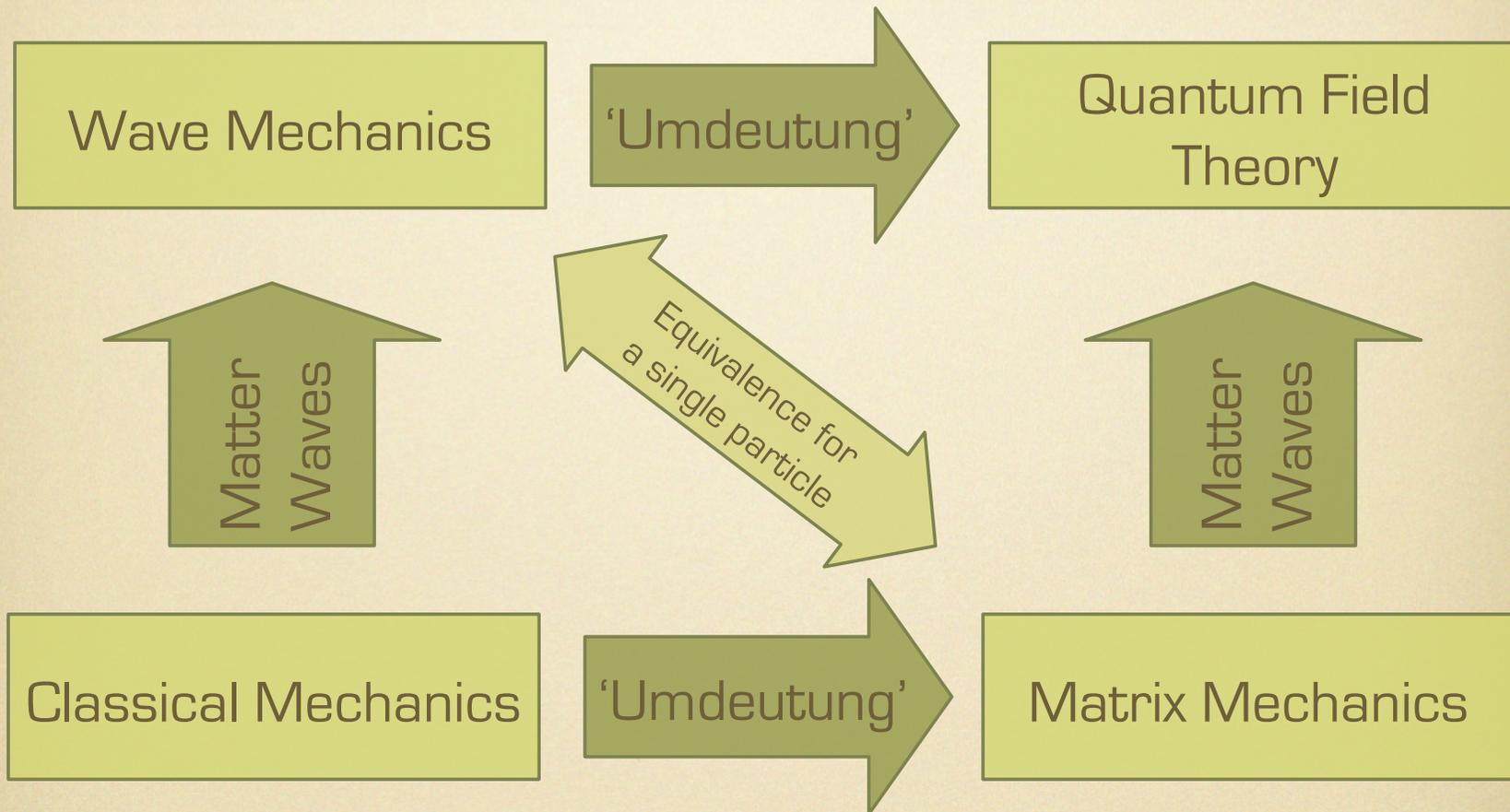
Quantum mechanics is only the first step to a full theory of matter, given by QFT of matter waves.

Quantum **mechanics** according to Jordan



You can take the second step before the first, since they are logically independent.

Quantum **mechanics** according to Jordan



But only for single particle states are the two incomplete theories equivalent.

Jordan's positivism

Unlike Schrödinger, however, Jordan does not believe that quantum field theory implies a priority of the field concept over the particle concept.

Quantized field is nothing like a classical field—it does not imply a continuous spatiotemporal picture.

Jordan maintains just like in transformation theory that both particle and wave description are complementary and cannot be unified into a single description of an objective reality.

The fundamental tension

Jordan's claims about the fundamental and **explanatory** role of quantum field theory are not easily reconcilable with his claims that transformation theory implies a strict positivism.

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Jordan's claims about the fundamental and **explanatory** role of quantum field theory are not easily reconcilable with his claims that transformation theory implies a strict positivism.

This correlates to a tension in the physical theory:

Transformation theory does not apply in the same sense to QFT as it does to quantum mechanics: QFT is built on an absolutely classical picture of spacetime.

Particle and wave representations of QFT are not related in a way analogous to the way transformation theory connects position and momentum eigenstates.

The problems of the program

Problem of infinities.

More important: No recognizable advantage in empirical predictions over Dirac's method of treating particles with quantum mechanics.

Dirac criticizes anticommutators as artificial at Solvay 1927, does not use QFT in his relativistic theory of the electron.

General doubts about the quantum field theory program.

Pauli becomes disenchanted again. Landau and Peierls doubt the measurability of field strengths. Fermi clarifies and popularizes Dirac's approach.

Both Heisenberg and Dirac attempt alternative formulations. Both turn out to be equivalent to the Heisenberg-Pauli approach.

The revolution eats its children

The development of QFT in the thirties show the problematic nature of positivism:

Unlike quantum mechanics, which has a solid mathematical basis, QFT has neither this nor a physical model to fall back on in the face of adversity.

QFT lacks the resources to deal with the problem of infinities, it does not develop new applications.

All the protagonists (Jordan, Dirac, Pauli, Heisenberg) abandon the program.

Feynman diagrams eventually come to the rescue and a new generation of physicists pick up the pieces.

Epilogue

Jordan never rejoined the forefront of research in quantum field theory. His early contributions were mostly forgotten by the time of QFT's renaissance after 1948, even though modern quantum field theory is closer to Jordan's program than to Dirac's original ideas.

However, also the problems are still there.

Literature

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Thanks

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