On an alleged fine-tuning problem in Dirac's electron theory and its partial solution by Weisskopf (and Furry)

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HQ3
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Murayama's two fine-tuning problems

Murayama:

$$m_e c^2 = (m_e)^0 c^2 + E_{self}. \tag{1}$$

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"As we reduce the "size" of the electron, the smaller we should take its "bare" mass $(m_e)^0$, maybe down to a negative value. It requires increasing fine-tuning to reproduce the observed electron mass."

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$$E_{self} = \frac{3}{4\pi} \frac{1}{4\pi\epsilon_0} \frac{e^2}{\lambda_C} \ln \frac{m_e c r_e}{\hbar}. \quad (2)$$

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kind of...

Murayama's two fine-tuning problems

"Even for a size equal to the Planck length [$\sim 10^{-35}$m], the self-energy is only about 10% correction to the 'bare' mass."

Weisskopf's early years

Victor Weisskopf, Maria Göppert-Mayer, and Max Born, n.d.; AIP Emilio Segre Visual Archives, Gift of Jost Lemmerich

Weisskopf's early years

- b. 1908, Vienna
- 1926–28: studied with Hans Thirring in Vienna
- 1928–31: studied at Göttingen (Born, Franck, Ehrenfest)
- 1931: Leipzig – postdoc with Heisenberg
- 1932 (winter): Berlin – assistant to Schrödinger
- 1932-33: Rockefeller fellowship
  - Copenhagen
  - Cambridge (Dirac, but really Peierls)
- 1933–36: Zurich – assistant to Pauli

Calculations of electron self-energy without hole theory

- R. Oppenheimer, (1930) "Note on the theory of the interaction between light and matter." *Phys. Rev.* 35: 461–77. (Received November 12, 1929)
- I. Waller, (1930) "Bemerkungen über die Rolle der Eignenergie des Elektrons in der Quantentheorie des Elektrons." *Z. f. Ph.* 62: 673–76. (Received 24 March 1930)
- L. Rosenfeld, (1931) "Zur Kritik der Diracschen Strahlungstheorie." *Z. f. Ph.* 70: 454–62.

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- L. Rosenfeld, (1931) "Zur Kritik der Diracschen Strahlungstheorie." *Z. f. Ph.* 70: 454–62.
- All of these used the Dirac theory applied to a single electron without consideration of the "sea" of occupied negative energy electron states.

Discovery of the positron

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Discovery of the positron

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From a Note in Phys Rev by Oppenheimer and Plesset (1933)

"The experimental discovery of the positive electron gives us a striking confirmation of Dirac's theory of the electron, *and of his most recent attempts to give a consistent interpretation of the formalism of that theory*. . . . Dirac has pointed out that we might obtain a consistent theory by assuming that it is only the absence of electrons of negative kinetic energy that has a physical meaning."

Basic ideas of hole theory

- The vacuum is a state in which all the negative energy levels of the Dirac equation are occupied.
- Transitions from positive to negative energy states are prevented by the Pauli exclusion principle.
- An unoccupied negative energy state will behave just like an electron in a positive energy state, but with positive charge (a positron).

With Hendrik Casimir in Copenhagen, 1934; photograph by Paul Ehrenfest Jr., AIP Emilio Segre Visual Archives, Weisskopf Collection.

Correspondence with Pauli

Pauli to Weisskopf, 2.8.1933

Pauli worried that, due to "a certain tendency to swing from one extreme to another," Bohr might enter a "period of extreme gullibility [Leichtgläubigkeit] in the face of hole ideas [Löcheridee]."

Correspondence with Pauli

Pauli to Weisskopf, 29.8.1933 on his "prophecy" regarding Bohr

In addition to his knowledge of Bohr's "Psychologie" Pauli concluded from a note of thanks to Bohr attached to the end of a Note by Oppenheimer (and Plesset) in Phys Rev that "everything that seemed alarming and dubious in the Note has its origin in Bohr."

Correspondence with Pauli

Pauli to Weisskopf, 29.8.1933

"Above all I do not believe him now as ever, that in the present state of the theory lengths of the order h/mc and those of the order of the classical electron radius $\frac{e^2}{m^2c^2}$ can be sharply divided, and even more that one can claim in regions of the order h/mc that everything is now in beautiful order. (That was more or less stated in the cited Note; what Bohr thinks *now* about it, I naturally do not know.)"

Correspondence with Pauli

Pauli to Weisskopf, 29.8.1933

"... a separation of the difficulties of the theory between those coming from questions of the stability of the electron and the atomism of the electric charge and those connected to the production and entanglement of particle pairs *is not at all possible*."

Correspondence with Pauli

Pauli to Weisskopf, 29.8.1933

Pauli acknowledges that due to the success of Dirac's "idea" at forecasting the existence of positive electrons, "Dann muß also auch etwas daran sein!" But "the problem of the quantitative elaboration of this idea into a theory seems insolubly tied up with the theoretical understanding of the atomism of electrical charge and the stability of the electron."

Weisskopf's 1934 paper

Über die Selbstenergie des Elektrons.

Von **V. Weisskopf** in Zürich.

(Eingegangen am 13. März 1934.)

Es wird die Selbstenergie des Elektrons im engeren formalen Anschluß an die klassische Strahlungstheorie abgeleitet, und die Selbstenergie eines Elektrons bei besetzten negativen Energiezuständen berechnet, was der Vorstellung der Diracschen „Löchertheorie" des positiven und negativen Elektrons entspricht. Wie zu erwarten, divergiert die Selbstenergie auch in dieser Theorie, und zwar in dem gleichen Grade wie in der gewöhnlichen Einelektronentheorie.

1. Problemstellung.

Die Selbstenergie des Elektrons ist die Energie des elektromagnetischen Feldes, das durch das Elektron erzeugt wird, zuzüglich der Energie der Wechselwirkung des Elektrons mit diesem Feld. Waller[1]), Oppenheimer[2]) und Rosenfeld[3]) berechneten die Selbstenergie des freien Elektrons mit Hilfe der Diracschen relativistischen Wellengleichung des Elektrons und der Diracschen Theorie der Wechselwirkung zwischen Materie und Licht. Sie benutzten dabei ein Näherungsverfahren, das die Selbstenergie

Z. f. Ph. 89: 27–39. Tr. in A. Miller, *Early Quantum Electrodynamics: A Source Book*

Weisskopf's 1934 paper

To calculate the electron self energy while taking account of negative energy states, Weisskopf uses, instead of direct application of QED, Heisenberg's 1931 radiation theory, which was advertised by Heisenberg as connecting more closely "*intuitive* conceptions of classical theory with those of wave mechanics" and yielding "in most cases, and without detours . . . the result expected in terms of the correspondence

*W. Heisenberg (1931). "Bemerkungen zur Strahlungstheorie." *Ann. der Phys.* 9: 23–66.

Weisskopf's 1934 paper

- For an electron with a surrounding electromagnetic field, the self energy is given by the expectation value of

$$E = -\int (\mathbf{iA}) d\mathbf{r} + \frac{1}{8\pi} \int (\mathbf{E}^2 + \mathbf{H}^2) \mathrm{d}\mathbf{r} \tag{3}$$

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  - "electrodynamic" E^D (from the "divergence-free" part with components $\mathbf{A}_{tr}, \mathbf{E}_{tr}, \mathbf{H}_{tr}$)
- Thus Weisskopf calculates the radiation field classically from current and charge densities, but modified to subtract out those due to the occupied negative energy states. In keeping with Heisenberg's radiation theory, the amplitudes of the potentials are regarded as non-commuting in the final result.

Weisskopf's 1934 paper

The result for E^S:

$$E^S = \frac{e^2}{h(p_0^2 + m^2c^2)}(2m^2c^2 + p_0^2) \cdot \int_0^\infty \frac{\mathrm{d}p}{p} + [\text{finite terms}] \quad (4)$$

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"The electrostatic self-energy thus diverges *logarithmically* in the 'hole' theory."

For comparison, in single-electron theory the divergence is linear:

$$E^S = \frac{e^2}{h} \int \mathrm{d}p + \text{finite terms} \quad (5)$$

Weisskopf's 1934 paper

For the electrodynamic contribution, Weisskopf has to take into account zero-point fluctuations and the non-commutability of the components of the vector potential. This leads, at length, to:

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$$E^D = \frac{11}{6}\frac{e^2}{hP}p^2\int\frac{dk}{k} - \frac{e^2}{h}\frac{m^2c^2}{pP}\log\frac{P+p}{P-p}\int \mathrm{d}k - \\ - \frac{2e^2}{hP}\int k\cdot \mathrm{d}k + \text{finite terms} \tag{6}$$

Where $k = |\mathbf{p}' - \mathbf{p}|$ and $P = +(p^2 + m^2c^2)^{1/2}$

Weisskopf's 1934 paper

For comparison, Weisskopf reiterated Waller's result for a single electron:

$$E^D = \frac{e^2}{h}\left[\frac{m^2c^2}{pP}\log\frac{P+p}{P-p} - 2\right]\int \mathrm{d}k + \frac{2e^2}{hP}\int k\mathrm{d}k \qquad (7)$$

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Both show quadratic divergence!

Weisskopf's 1934 paper

"Pauli told me that we must re-calculate the self energy of the electron because so far it had only been calculated by Ivar Waller for a single particle, that is, not with the Dirac theory of the vacuum. 'The vacuum will probably change it; try to calculate it,' Pauli said. So I sat down to calculate it and made a calculation mistake, a very primitive, bad mistake, which made me terrifically unhappy at the time, and got the result that it diverges as badly as Waller's diverges." (AHQP interview by Kuhn and Heilbron)

Letter from Furry

Wendell Furry in 1941; photograph by Samuel Goudsmit, courtesy AIP Emilio Segre Visual Archives, Goudsmit Collection

Letter from Furry

"Dear Dr. Weisskopf:
The *ZS. f. Physik* containing your paper on the proper energy of the electron arrived here some days ago. Prof. Oppenheimer and I were much interested in the method you followed, and note that it is in complete agreement with the methods of 'pair theory', as we formulated it in our *Physical Review* paper. . . . We are, however, not able to agree with your result for the magnetic proper energy. About a year ago at Prof. Bohr's suggestion Dr. Carlson and I made this calculation. Our result was

$$E^D = \frac{mc}{(m^2c^2 + p^2)^{1/2}}\left[1 - \frac{4}{3}\frac{p^2}{m^2c^2}\right] \cdot \frac{e^2}{hc} \int \frac{\mathrm{d}k}{k} + \text{finite terms} \quad (8)$$

and is of same order as your result for the electrostatic proper energy."

Letter from Furry

"…the contribution to E^D from the negative K.E. states is much *smaller* than in the calculation with empty negative K.E. states, instead of merely being opposite in sign. This is physically evident when we use the light quantum picture and treat the energy displacement as due to the transitions of the system from a given state to other states and back, with corresponding emission and reabsorption of light quanta. For in the case of transition to a state of large negative K.E. and simultaneous emission of a quantum there is a high degree of resonance, and in the case of simultaneous pair production and light quantum emission such a resonance is lacking."

Pauli's response

"[Pauli's] attitude was, 'Well, I didn't expect much more from you; I'm not surprised. I never make mistakes, but of course you people do. That happens when one has to work with people like this.' And, 'I *said* that it must be different'. "
(AHQP interview by Kuhn and Heilbron)

The error corrected

When he corrected the error, Weisskopf obtained in place of (6) for hole theory:

$$E^D = \frac{e^2}{h(m^2c^2 + p^2)^{1/2}}(m^2c^2 - \frac{4}{3}p^2) \int \frac{\mathrm{d}k}{k} + + \text{finite terms} \quad (9)$$

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and in place of (7) for single-electron theory:

$$E^D = \frac{e^2}{h}\Big[\frac{m^2c^2}{p(m^2c^2+p^2)^{1/2}}\log\frac{(m^2c^2+p^2)^{1/2}+p}{(m^2c^2+p^2)^{1/2}-p} - 2\Big]\int \mathrm{d}k + \\ + \frac{2e^2}{h(m^2c^2+p^2)^{1/2}}\int k\mathrm{d}k \tag{10}$$

Some milestones, 1934–1939

- 1934: Pauli and Weisskopf, "Über die Quantisierung der skalaren relativistischen Wellengleichung." *Helv. Phys. Acta* 7: 709–31 ("anti-Dirac" for Bose-Einstein particles; production and annihilation processes without hole theory)

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- 1936: Weisskopf, "Über die Elektrodynamik des Vakuums auf Grund der Quantentheorie des Elektrons." *Det Kongelige Danske Videnskaberes Selskab. Mathematisk-fysiske Meddeleser* 14(6): 3–39 (uses Dirac's positron theory with negative energy states to identify divergent properties of vacuum electrons; argues that these may be regarded as physically meaningless, motivating "subtraction physics")

Some milestones, 1934–1939

- 1938: Dirac, "Classical theory of radiating electrons." *Proc. Roy. Soc. (A)* 167: 148–69 (renormalized classical electrodynamics before it was accomplished for QED (Rohrlich, 1960); deals with infinite energies "by a process of direct omission or subtraction of unwanted terms, somewhat similar to what has been used in the theory of the positron")

Unresolved questions

. . . I felt this guilt that we do not understand the self energy and this logarithmic self energy. I have to really investigate thoroughly where this self energy comes from. Then I wrote this paper in the *Physical Review* of '39 . . . It investigated all the details of why the self energy is logarithmic, how it comes about that the influence of the vacuum on the electron makes the electron broader and therefore increases the Coulomb energy, and all these detailed analyses of what's going on."
(AHQP interview by Kuhn and Heilbron)

Weisskopf 1939

JULY 1, 1939 PHYSICAL REVIEW VOLUME 56

On the Self-Energy and the Electromagnetic Field of the Electron

V. F. Weisskopf
University of Rochester, Rochester, New York

(Received April 12, 1939)

The charge distribution, the electromagnetic field and the self-energy of an electron are investigated. It is found that, as a result of Dirac's positron theory, the charge and the magnetic dipole of the electron are extended over a finite region; the contributions of the spin and of the fluctuations of the radiation field to the self-energy are analyzed, and the reasons that the self-energy is only logarithmically infinite in positron theory are given. It is proved that the latter result holds to every approximation in an expansion of the self-energy in powers of e^2/hc. The self-energy of charged particles obeying Bose statistics is found to be quadratically divergent. Some evidence is given that the "critical length" of positron theory is as small as $h/(mc)\cdot\exp(-hc/e^2)$.

Weisskopf 1939

On the self-energy and the electromagnetic field of the electron

"The main purpose of this paper is to show the physical significance of the logarithmic divergence and to demonstrate the reasons of its occurrence" according to positron theory.

Weisskopf 1939

Three reasons why quantum theory of the electron "has put the problem of the self-energy in a critical state":

1. Quantum kinematics demands that the radius a of the electron must be assumed to be zero, and the electrostatic field self-energy contribution is infinite as $W_{st} = \lim_{(a=0)} e^2/a$.

Weisskopf 1939

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1. Quantum kinematics demands that the radius a of the electron must be assumed to be zero, and the electrostatic field self-energy contribution is infinite as $W_{st} = \lim_{(a=0)} e^2/a$.
2. The relativistic quantum theory of the electron attributes to it a magnetic moment due to spin; this means there is a magnetic field contribution to the self-energy $U_{mag} \propto a^{-3}$. However this is canceled by the contribution of an alternating electric field due to "Zitterbewegung".

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2. The relativistic quantum theory of the electron attributes to it a magnetic moment due to spin; this means there is a magnetic field contribution to the self-energy $U_{mag} \propto a^{-3}$. However this is canceled by the contribution of an alternating electric field due to "Zitterbewegung".
3. QT of the electromagnetic field postulates fluctuations in empty space. These contribute an additional energy that diverges quadratically as $a \to 0$.

Weisskopf 1939

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Consider one electron in vacuum:

Weisskopf 1939

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- vacuum: all negative energy states filled

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Consider one electron in vacuum:

- vacuum: all negative energy states filled
- only *difference* between "actual" and unperturbed charge density of vacuum is observable
- effect of Pauli principle "similar to a repulsive force" between particles with equal spin at distances $\sim h/p$

Weisskopf 1939

- "As a consequence of this we find at the position of the electron a 'hole' in the distribution of the vacuum electrons which completely compensates its charge."

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- Also a " cloud of higher charge density coming from the displaced electrons, which must be found one wave-length from the original electron."

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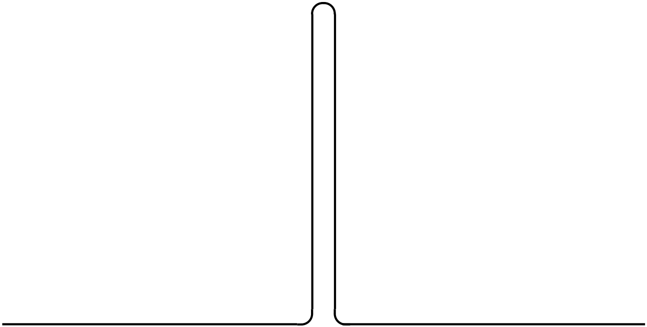
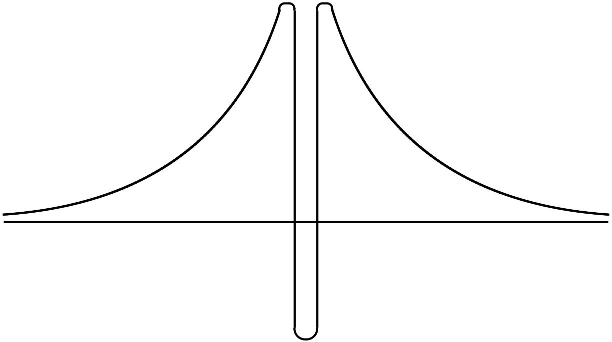
- "As a consequence of this we find at the position of the electron a 'hole' in the distribution of the vacuum electrons which completely compensates its charge."
- Also a " cloud of higher charge density coming from the displaced electrons, which must be found one wave-length from the original electron."
- Thus there is a "broadening" of electron's charge over $\sim h/mc$.

Weisskopf 1939



Schematic charge distribution of the electron

Weisskopf 1939



Schematic charge distribution of the vacuum electrons in the neighborhood of an electron

Weisskopf 1939

- Weisskopf shows that this reduces the electrostatic self-energy to a logarithmic divergence.

Weisskopf 1939

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- This broadening of the electron charge also changes how the magnetic field energy is distributed. The magnetic field energy is equal to that of a momentum distribution spread over a finite region and is given by:

$$W_{sp} = -\lim_{(a=0)}\left[\frac{e^2 h}{2\pi mca^2} - \frac{e^2 mc}{4\pi h}\cdot\log\frac{h}{mca}\right] \qquad (11)$$

Weisskopf 1939

- However, the electric field energy of the spin no longer cancels that of the magnetic field contribution. This is because vacuum electrons fluctuate with phase opposite that of original electron. This produces an interference effect that reduces the total electric field energy.

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- In other words, the contribution of the electron itself – considered as the difference between the field energy of the vacuum and the field energy of the vaccum + 1 electron system – to the total electric field energy due to spin is *negative* . . .

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$$W_{sp} = -2U_{mag} = -\lim_{(a=0)}\left[\frac{e^2 h}{\pi mca^2} - \frac{e^2 mc}{2\pi h}\cdot\log\frac{h}{mca}\right] \quad (12)$$

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- Effect of field on displaced vacuum electrons is negligible, so to first approximation, W_{fluct} is the same in positron theory as in single electron theory, and thus diverges quadratically . . .
- . . . and thus cancels the quadratic term of the divergence in W_{sp}.

Weisskopf 1939

Three parts to electron self-energy in positron theory:

a. Energy of Coulomb field W_{st} – log divergence

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a. Energy of Coulomb field W_{st} – log divergence
b. Energy of "oscillatory motion which produces spin" W_{sp} – quadratic + log divergence
c. Energy of "forced vibrations under the influence of the zero-point fluctuations of the radiation field" W_{fluct} – quadratic divergence

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Three parts to electron self-energy in positron theory:

a. Energy of Coulomb field W_{st} – log divergence
b. Energy of "oscillatory motion which produces spin" W_{sp} – quadratic + log divergence
c. Energy of "forced vibrations under the influence of the zero-point fluctuations of the radiation field" W_{fluct} – quadratic divergence

- Contributions from b and c are opposite in sign, so that only the logarithmic divergence is left.

Summary

- Negative energy states do a play role in Weisskopf 1934, but their physical interpretation does not
- Weisskopf 1934 has almost no physical reasoning of any kind (a reason for his failing to catch his computational error?)
- Weisskopf 1939 relies heavily on the hole-theoretic interpretation of QED
- Perhaps he draws confidence from earlier success in defending subtraction physics as physically principled?

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