The Overlapping Worlds of General Relativity and Quantum Theory: The Challenge of the Principle of Equivalence

Jürgen Renn, Donald Salisbury, Matthias Schemmel, and Kurt Sundermeyer (MPIWG, Austin College, MPIWG, FU Berlin)

July 1, 2010

Overview

- The long history of quantum gravity
- A historical-critical approach
- The overlapping worlds of relativity and quantum mechanics
- From the principle of equivalence to Einstein's 1912 classical particle dynamics in a static gravitational field
- Pathways to the Schrödinger equation in locally accelerated frames
- Outlook

The long history of quantum gravity

- Recognition of need to reconcile gravitational and quantum physics by Einstein already in 1916
- Canonical approaches to quantum gravity: DeDonder (1926), Klein (1927), Rosenfeld (1930), Bronstein (1936), Bergmann (1949), Dirac (1958), Arnowitt, Deser and Misner (1962)
- Covariant quantization: DeWitt (1950), Feynmann (1962), Gupta (1962)
- Early conferences: Warsaw (1938), Bern (1955), Chapel Hill (1957), Warsaw (1962)
- Entrenched alternatives: string theory, loop quantum gravity, non-commutative geometries, $\cdots$

A historical-critical approach

- The role of Mach's historical-critical analysis in the genesis of general relativity
- The role of Hamilton's optical-mechanical analogy in the emergence of wave mechanics
- Lessons from the Newstein fiction in elucidating the historiographic significance of post-mature and pre-mature discoveries
- The unlikely historical precedence of general relativity over quantum mechanics

The overlapping worlds of relativity and quantum mechanics

- Interdependence of matter and spacetime in the relativity and quantum theories: new description of matter by stress energy tensor; non-locality of quantum mechanics; need of physical fields to construct geometry
- Clashes of principles: violation of uncertainty principle through classical character of gravitation? validity of equivalence principle in quantum mechanics?
- Borderline problems: black holes, early universe
- The interaction between representation and conceptualisation: path integral formulation of quantum gravity; non-commutative geometry; twistors; string theory

Einstein's 1912 classical particle dynamics in a static gravitational field

- Building blocks
  - Newtonian dynamics, special theory of relativity, equivalence principle
- Early insights
  - light bending, gravitational redshift, representation of gravitational potential by variable speed of light
- Goals
  - Find particle equations of motion and gravitational field equations
- Main results
  - Metric represents gravitational fields
  - Identification of dynamical quantities: energy, inertial and gravitational mass
  - Geodesic equations of motion

Einstein's transformation to an accelerated frame

Let ξ and τ represent Minkowski coordinates, then the Einstein transformations to the accelerated frame are

$$\xi = x + \frac{1}{2}act^2,$$

and

$$\tau = \frac{c}{C}t,$$

where

$$c := c_0 + ax \tag{1}$$

is the variable speed of light. This speed served a dual role in determining the time elapsed by unaltered clocks: c^2dt^2

Conservation law of the new dynamics

Einstein showed that the following quantity is conserved:

$$e = \frac{c}{\sqrt{1 - \frac{q^2}{c^2}}}, \tag{2}$$

where he defines

$$q^2 := \left(\frac{d\vec{x}}{dt} \cdot \frac{d\vec{x}}{dt} \right). \tag{3}$$

Relation of Einstein to Rindler transformations

$$\xi' = \rho\, cosh(at), \tag{4}$$

and

$$C\tau = \rho\, sinh(at). \tag{5}$$

Recall that $\rho = C^2/A$, where A is the proper acceleration.
We let $\rho = \rho_0 + x$, and keep terms up to the second order in t. So let

$$\xi = (\rho_0 + x)\left(1 + \frac{a^2t^2}{2}\right) - \rho_0, \tag{6}$$

and

$$C\tau = (\rho_0 + x)at. \tag{7}$$

Small deviation from Minkowski spacetime

Now define

$$CdT = \rho_0 a dt, \tag{8}$$

or, recalling that $\rho = C^2/A$,

$$dT = \frac{Ca}{A_0} dt. \tag{9}$$

This gives the line element

$$ds^2 = \left(1 + 2\frac{A_0 x}{C^2}\right) C^2 dT^2 - dx^2 - dy^2 - dz^2. \tag{10}$$

Then the speed squared is

$$v^2 := \left(\frac{d\vec{x}}{dT} \cdot \frac{d\vec{x}}{dT}\right) = \frac{A_0^2}{C^2 a^2} q^2,$$

The relativistic energy

Under these assumptions Einstein's conserved quantity is proportional to the relativistic energy E

$$E = \frac{A_0}{a} me = mC^2 \left(1 + \frac{v^2}{2C^2} + \frac{v^2}{2C^2} \frac{A_0 x}{C^2} \right) \tag{11}$$

The relativistic energy (and mass) exhibits a kinematic and gravitational increase. (Note that as a consequence of the presumed equality of inertial and gravitational rest mass, the gravitational mass undergoes both a kinematic and gravitational increase.)

Kiefer procedure - 1994

The only non-vanishing Christoffel coefficient is

$$\Gamma^1_{00} = \frac{1}{2}\left(1 + \frac{2\zeta A_0}{C^2}\right) g_{00,1} = \left(1 + \frac{\zeta A_0}{C^2}\right)\frac{A_0}{C^2}. \tag{12}$$

Therefore the minimally coupled Klein-Gordon equation is

$$\begin{aligned}
\phi^{;\mu}{}_{\mu} + \frac{m^2 C^2}{\hbar^2}\phi &= \left(1 - \frac{2\zeta A_0}{C^2}\right)\left(\frac{1}{C^2}\ddot{\phi} - \Gamma^1_{00}\phi_{,1}\right) - \nabla^2\phi + \frac{m^2 C^2}{\hbar^2}\phi \\
&= \left(1 - \frac{2\zeta A_0}{C^2}\right)\left(\frac{1}{C^2}\ddot{\phi} - \frac{A_0}{C^2}\left(1 + \frac{2\zeta A_0}{C^2}\right)\phi_{,1}\right) \\
&- \nabla^2\phi + \frac{m^2 C^2}{\hbar^2}\phi.
\end{aligned} \tag{13}$$

C^{-2n} expansion

Now we construct the solution for ϕ in inverse powers of C, letting

$$\phi = e^{iS/\hbar}, \tag{14}$$

and expanding S as

$$S = C^2 S_0 + S_1 + C^{-2} S_2 + \cdots \tag{15}$$

Summary of results through C^0

- C^4: $\vec{\nabla} S_0 = 0$.
- C^2: $\dot{S}_0^2 = m^2$. Take $S_0 = -m$.
- C^0: Define $\chi := e^{iS_1/\hbar}$. Then

$$-\frac{\hbar^2}{2m}\nabla^2\chi + mA_0\zeta\chi = i\hbar\dot{\chi}, \tag{16}$$

Order C^{-2}: Toward a quantum particle dynamics in a static gravitational field

$$
\begin{aligned}
0 &= \frac{i}{\hbar}\ddot{S}_1 - \frac{1}{\hbar^2}\left(\dot{S}_1^2 - 2m\dot{S}_2\right) - \frac{4m}{\hbar^2}xA_0\dot{S}_1 - \frac{i}{\hbar}\nabla^2 S_2 - A_0\frac{i}{\hbar}S_{1,1} \\
&+ \frac{2}{\hbar^2}\vec{\nabla}S_1 \cdot \vec{\nabla}S_2\phi + \frac{m^2C^2}{\hbar^2}\phi. \qquad (17)
\end{aligned}
$$

Define $\psi := \chi e^{i\frac{S_2}{\hbar C^2}}$, then

$$
\begin{aligned}
i\hbar\dot{\psi} &= -\frac{\hbar^2}{2m}\nabla^2\psi + mA_0x\psi - \frac{\hbar^4}{8m^3C^2}\nabla^2\nabla^2\psi \\
&- \frac{\hbar^2}{2mC^2}A_0x\nabla^2\psi. \qquad (18)
\end{aligned}
$$

Canonical expression of Einstein energy

Solve

$$\vec{p} = \frac{E}{C^2}\vec{v},$$

for $\frac{v^2}{C^2}$. Yields

$$\frac{v^2}{C^2} = \frac{p^2}{m^2 C^2}\left(1 + \frac{A_0 x}{C^2} - \frac{p^2}{m^2 C^2}\right) \tag{19}$$

Therefore

$$E = mC^2\left(1 + \frac{xA_0}{C^2} + \frac{p^2}{2m^2 C^2} + \frac{xA_0}{C^2}\frac{p^2}{m^2 C^2} - \frac{p^4}{8m^4 C^2}\right) \tag{20}$$

In terms of Einstein's variable light speed

$$E = A_0 mc\left(1 + \frac{p^2}{m^2 C^2} - \frac{p^4}{8m^4 C^2}\right) \tag{21}$$

Counterfactual history?

DeDonder (1926)

$$(3) \quad \sum_\alpha \sum_\beta g^{\alpha\beta}(p_\alpha - e'\Phi_\alpha)(p_\beta - e'\Phi_\beta) - (c^2 m')^2 = 0 \qquad (\alpha, \beta = 1, 2, 3, 4).$$

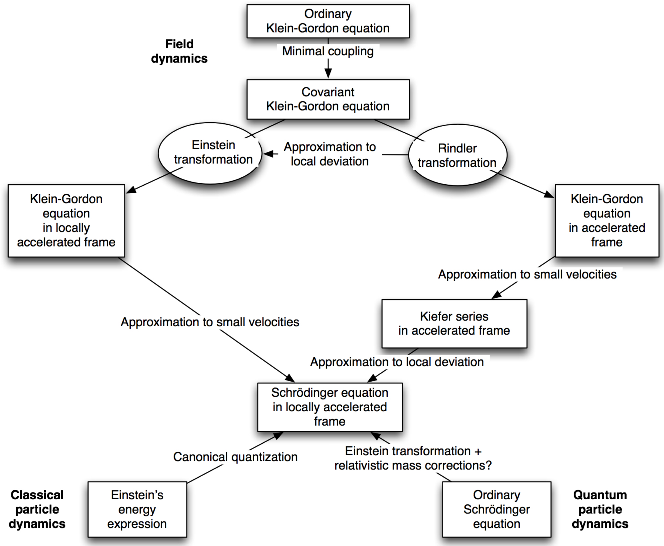
tion de Jacobi, en posant $k\mathrm{S} = ln\,\Psi$,

$$(4) \quad \mathrm{I} \equiv \sum_i \sum_j g^{ij}\left(\frac{\partial\Psi}{\partial x_i} - e'\Phi_i k\Psi\right)\left(\frac{\partial\Psi}{\partial x_j} - e'\Phi_j h\Psi\right) + 2k\Psi \sum_i g^{4i}(c\mathrm{E} + c^3 m' - e'\Phi_4)\left(\frac{\partial\Psi}{\partial x_i} - e'\Phi_i k\Psi\right) + k^2\Psi^2[g^{44}(c\mathrm{E} + c^3 m' - e'\Phi_4)^2 - (c^2 m')^2] = 0$$
$$(i, j = 1, 2, 3).$$

Comme *exemple*, prenons *un champ de Minkowski* dans lequel l'électron se meut ; l'équation (5) devient :

$$(7) \quad \Delta\Psi - k\Psi\left[e'\left(ke'\mathcal{C}\frac{1}{c^2}\frac{\partial\Phi_4}{\partial t} + k2m'e^2\left(\mathrm{E} - \frac{e'\Phi^4}{c}\right) + k\left(\mathrm{E} - \frac{e'\Phi_4}{c}\right)^2\right]\right. = 0.$$

Pathways toward relativistic quantum particle dynamics in a static gravitational field



Outlook

- Coupling to plane wave - harmonic oscillator
    - Link to Unruh effect and thermodynamics?
- Hamilton-Jacobi ray limit of Einstein particle dynamics
- Recovery of static potential in semi-classical Hamilton-Jacobi limit of Wheeler-DeWitt equation
- Much more!