The Transformation of Mechanics

Jürgen Renn, Christian Joas, Christoph Lehner
Max Planck Institute for the History of Science, Berlin

Berlin, June 28, 2010
Outline

- **Revolution or Transformation?** The fate of the knowledge of classical physics.
- **Dizygotic twins?** The quantum revolution and the two versions of the new mechanics.
- **Classical Roots?** The refinement of the correspondence principle vs. the optical-mechanical analogy.
Part I: Revolution or Transformation?
Challenges to the mechanical worldview arise at the borderline between these theories!
Revolution or Transformation?

- Three major new conceptual frameworks emerge at the beginning of the 20th century:
  - quantum physics
  - relativity physics
  - statistical physics

- Where did the knowledge come from that enabled the development of these frameworks?

- Which role did previously established knowledge play?
The Relativity Revolution

- The **paradox of missing knowledge**: Few empirical hints towards a theory radically different from Newton’s mechanics.

- Historical research has shown: Relativity theory was a **transformation of classical physics** resulting from a **reorganization** of established knowledge under new principles.

- For example: Re-interpreting inertial forces as the effects of a generalized gravito-inertial field (**Equivalence Principle**).
The Origins of the Quantum Revolution

Mechanics

Conflicts with new empirical evidence:
- black-body radiation
- atomic spectra
- specific heat
- X-ray absorption
- Stern-Gerlach experiment

Borderline problems with:
- electrodynamics
- thermodynamics
- chemistry

Quantum Revolution
Quantum vs. Relativity Revolution

- Few **actors** in relativity vs. many in quantum.
- Scarce **empirical basis** in relativity vs. a bulk of new empirical findings in quantum.
- One final **formulation** in relativity vs. two distinct formulations in quantum: matrix and wave mechanics.
Old Quantum Theory

- The old quantum theory consisted in augmenting Hamiltonian mechanics by auxiliary conditions.

- **Quantum condition**: The action integral around a classical orbit must be an integer multiple of Planck’s quantum of action:
  \[ \int pdq = nh \]

- **Correspondence principle**: The classical theory of electrodynamics offers a limit which restricts possible transitions between orbits.

- These were **heuristic schemes** rather than full-fledged theory.

- What were the crucial steps in the transition from old quantum theory to either matrix or wave mechanics?
The Crisis of the Old Quantum Theory

- The old quantum theory **failed** to explain many empirical findings: Helium spectrum, Zeeman effect, multiplet structure of atomic spectra, aperiodic phenomena in general.

- From ca. 1923, **doubts** in the validity of the scheme of old quantum theory arose.

- Instead of a heuristic scheme, physicists now sought for a “sharpened” formulation of the **correspondence principle** that would yield a full theory with the explanatory power to tackle the open problems.

- Heisenberg’s 1925 **matrix mechanics** was an attempt to accomplish this using insights from the problems that troubled the old quantum theory (e.g., dispersion, multiplet structure).

- In 1926, Schrödinger’s **wave mechanics**, however, offered an equally general theory, based on rather different evidence and principles.

- Very rapidly, it became clear that the two new theories are essentially **equivalent**.

- **How can this be?**
Part II: Dizygotic Twins?
Two New Versions of Mechanics

- Which knowledge enabled the crucial step to the two new versions of mechanics?
- How could there be two distinct approaches to what later turned out to be equivalent in important respects?
- Why was the reformulation of Bohr’s correspondence principle crucial for one theory and immaterial for the other?
Candidates for Knowledge Fueling the Crucial Step towards Quantum Mechanics

- 1900 Planck’s radiation formula for heat radiation with the help of the energy-frequency relationship
- 1905 Einstein’s explanation of the photoelectric effect with the help of the light quantum hypothesis
- 1913 Bohr’s explanation of the hydrogen spectrum with the help of his atomic model
- 1916 Schwarzschild’s and Epstein’s explanation of the Stark effect with the help of a modified Hamiltonian mechanics
- 1916 Einstein’s derivation of the black-body radiation formula from the Bohr model with the help of emission and absorption coefficients
- 1923 de Broglie’s explanation of Bohr’s quantum conditions using a wave theory of matter
- 1924 Kramers’ and Heisenberg’s explanation of optical dispersion with the help of the correspondence principle
- 1924 Einstein’s and Bose’s explanation of Nernst’s heat theorem with the help of a new statistics
Knowledge Fueling the Crucial Step towards Matrix Mechanics

- 1900 Planck’s radiation formula for heat radiation with the help of the energy-frequency relationship
- 1905 Einstein’s explanation of the photoelectric effect with the help of the light quantum hypothesis
- 1913 Bohr’s explanation of the hydrogen spectrum with the help of his atomic model
- 1916 Schwarzschild’s and Epstein’s explanation of the Stark effect with the help of a modified Hamiltonian mechanics
- 1916 Einstein’s derivation of the black-body radiation formula from the Bohr model with the help of emission and absorption coefficients
- 1923 de Broglie’s explanation of Bohr’s quantum conditions using a wave theory of matter
- 1924 Kramers’ and Heisenberg’s explanation of optical dispersion with the help of the correspondence principle
- 1924 Einstein’s and Bose’s explanation of Nernst’s heat theorem with the help of a new statistics
Knowledge Fueling the Crucial Step towards Wave Mechanics

- 1900 Planck’s radiation formula for heat radiation with the help of the energy-frequency relationship
- 1905 Einstein’s explanation of the photoelectric effect with the help of the light quantum hypothesis
- 1913 Bohr’s explanation of the hydrogen spectrum with the help of his atomic model
- 1916 Schwarzschild’s and Epstein’s explanation of the Stark effect with the help of a modified Hamiltonian mechanics
- 1916 Einstein’s derivation of the black-body radiation formula from the Bohr model with the help of emission and absorption coefficients
- 1923 de Broglie’s explanation of Bohr’s quantum conditions using a wave theory of matter
- 1924 Kramers’ and Heisenberg’s explanation of optical dispersion with the help of the correspondence principle
- 1924 Einstein’s and Bose’s explanation of Nernst’s heat theorem with the help of a new statistics
Distinct Knowledge Resources for Matrix and Wave Mechanics?

- Crossover Phenomenon:
  - Wave mechanics grew out of attempts to explain the hydrogen spectrum and covered optical dispersion only in the aftermath.
  - Matrix mechanics grew out of attempts to explain optical dispersion dispersion and covered the hydrogen spectrum only in the aftermath.

- How could wave mechanics come ultimately to the same conclusions as matrix mechanics without dispersion theory as an ingredient?
Pre-established Harmony: Possible reasons?

- Was wave mechanics just a **re-dressing** of matrix mechanics which already was known to Schrödinger?
- Were both theories **incomplete** and did only their synthesis give rise to what we today know as quantum mechanics?
- Does reality enforce **convergence** of different theoretical approaches?
- Were pre-existing **mathematical structures**, such as the Hilbert space formalism, uncovered independently by the two approaches?
Part III:
Classical Roots
The Search for a “Sharpening” of the Correspondence Principle

- Around 1924, attempts were made to “sharpen” the **correspondence principle** into a general translation procedure allowing to derive quantum states from a classical description of physical systems.

- e.g., Born’s discretization of differential equations in his 1924 article “Über Quantenmechanik.”

- The successful application of virtual oscillators in the context of dispersion served as a hint that they might be a **model base different from classical orbits** for such a sharpened correspondence principle.
Heisenberg 1925: Umdeutung

The basic idea is: In the classical theory, knowing the Fourier expansion of the motion is enough to calculate everything, not just the dipole moment (and the emission), but also the quadrupole and higher moments, etc.

Heisenberg to Kronig, May 1925
Heisenberg, Kramers (Jan. 1925): Dispersion Theory

- Dispersion determined by dipole moment of the field
- Fourier transform of the orbit
- Orbit $x(t)$
- Correspondence principle
- Ersatz oscillators
- Dispersion determined by dipole moment of the field
The Search for the Sharpened Correspondence Principle

all physical effects

Fourier transform of the orbit

orbit x(t)

Ersatz oscillators

correspondence principle

?
Heisenberg (July 1925):
Umdeutung

- orbit $x(t)$
- Fourier transform of the orbit
- all physical effects determined by multipole expansion of the field

- correspondence principle

- Umdeutung

- all physical effects determined by algebraic expressions of amplitudes
- Ersatz oscillators
- array of amplitudes (x-matrix)
Heisenberg’s Re-Casting of the Correspondence Principle

\[ B_\beta(n) e^{i\omega(n)\beta t} = \sum_{-\infty}^{+\infty} a_\alpha a_{\beta-\alpha} e^{i\omega(n)(\alpha + \beta - \alpha)t} \]

\[ = \int_{-\infty}^{+\infty} a_\alpha a_{\beta-\alpha} e^{i\omega(n)(\alpha + \beta - \alpha)t} d\alpha, \]

Quantentheoretisch scheint es die einfachste und natürlichste Annahme, die Beziehungen (3, 4) durch die folgenden zu ersetzen:

\[ B(n, n - \beta) e^{i\omega(n, n-\beta)t} = \sum_{-\infty}^{+\infty} a_\alpha a_{n-\alpha} a_{n - \alpha, n - \beta} e^{i\omega(n,n-\beta)t} \]

\[ = \int_{-\infty}^{+\infty} d\alpha a_\alpha a_{n-\alpha} a_{n - \alpha, n - \beta} e^{i\omega(n,n-\beta)t}; \]

und zwar ergibt sich diese Art der Zusammensetzung nahezu zwangläufig aus der Kombinationsrelation der Frequenzen.
Schrödinger 1926: Wave Mechanics

- Schrödinger found a “wave” generalization of Hamiltonian mechanics through the optical-mechanical analogy.
- This led him to his new mechanics.
- This also explains Schrödinger’s later stance on interpretation.

The Hamiltonian analogy in Schrödinger’s notebook [ca. 1918–1920].
Hamilton’s Optical-Mechanical Analogy

abstract attempt at unifying optics and mechanics
Schrödinger’s Completion of Hamilton’s Analogy
Old quantum theory is the limiting case of a more general wave mechanics!
Schrödinger’s Counterpart to the Sharpened Correspondence Principle

[...] the central claim of quantum theory appears to consist in the fact that the constant K has the universal value

\[ \frac{\hbar}{2\pi} \]

i.e., that the wave motion in the q-space constructed with this value of the constant has a real significance.
Conclusion: Pre-established Harmony?
The Genetic View:

- Both theories are transformations of a common ancestor: old quantum theory!
- Both theories preserve the formal structure of Hamiltonian mechanics, while extending just to the right degree.
- Both theories involve a translation procedure connecting classical with quantum concepts.
- Both theories incorporate the new knowledge about the energy-frequency condition.